Natural Resource Trade under the Threat of War*

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Abstract

We consider a dynamic environment in which a resource-rich country trades an exhaustible resource with a resource-poor country. In every period, the resource-poor country can arm and attack the resource-rich country. When the resource is extracted by price-taking firms, each firm fails to internalize the impact of its extraction on military action by the resource-poor country. In the empirically relevant case of inelastic resource demand, war incentives increase over time. If the resource-poor country is sufficiently strong, war becomes inevitable, encouraging more rapid extraction, and inducing war to happen earlier. We explore the extent to which this externality across price-taking firms can be internalized by the government of the resource-rich country regulating the price and the level of resource production. Two new economic forces emerge in such an environment. On the one hand, the resource-rich country's ability to control production can prevent war, and this occurs through a deviation from the Hotelling rule. For instance, if demand is inelastic, a slower extraction of resources than under the Hotelling rule emerges as a way of reducing armaments and incentives for war. On the other hand, the resource-rich country's inability to commit to attractive offers requires the resource-poor country to arm even if it is not fighting in order to guarantee the offer. If this cost of continually arming is sufficiently high, then war can be made inevitable in such a setting even when it could be prevented in an environment with price-taking firms.

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Preliminary and Incomplete. Comments Welcome.

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1 Introduction

> "Throughout human history, people and countries have fought over natural resources. [...] [T]oday, the uninterrupted supply of fuel and minerals is a key element in geopolitical considerations. Things are easier in times of plenty, when all can share in the abundance, even if to different degrees. But when resources — whether energy, water or arable land — are scarce, our fragile ecosystems become strained, as do the coping mechanisms of groups and individuals. This can lead to a breakdown of established codes of conduct, and even to outright conflict." –Ban Ki-moon, UN Secretary-General (2007).
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> "The oil market will remain fairly stable, but with steadily increasing prices as world production peaks. Demand now exceeds production and we are seeing that effect on prices. After the peak is reached, geopolitics and market economics will result in significant price increases above what we have seen to date. Security risks will also rise. To guess where this is all going to take us is would be too speculative. Oil wars are certainly not out of the question." –Report of the US Army Corps of Engineers (2005).

International trade and military conflict have been entwined throughout history (e.g., Findlay and O'Rourke, 2007). Several instances of international expansions, including the founding of European colonial empires starting in the 15th century, were motivated by the desire to have access to valuable resources. But what ensued was not "free trade," rather trade with the terms of trade being heavily influenced by military threats and actions. The Dutch East Indies Company, for example, used its own private army in the 16th and 17th centuries to monopolize the trade of nutmeg, clove, mace and pepper in the Indian Ocean, in many cases resorting to force and even to systematic killings in order to acquire these spices at low prices (Lieberman, 2003, van Zanden, 1993). The English East India Company, and subsequently England, followed the same model in its colonization of the Indian subcontinent. The 'Scramble for Africa' in the 19th century was also motivated by the desire of several European powers to forcibly gain access to resources on favorable terms of trade.

While there is a greater separation between international trade and military relations today, the shadow of military threat is not entirely absent, particularly in the context of key exhaustible resources such as oil. The strategic and military importance of the Middle East stems in large part from its disproportionate role in the supply of oil. Many commentators and politicians believe that the growing scarcity of oil in the coming decades will intensify conflict and perhaps even cause global wars.

In this paper, we take a first step in the analysis of the dynamic interactions between economic equilibria and inter-country military actions, focusing on the relationship between trade in exhaustible resources and war. We study a world economy consisting of two countries. The entire stock of an exhaustible resources is located in country S, and is demanded by country A.

A second (non-resource) consumption good can be used to make transfers from country A to country S. Throughout, we refer to this good as the "consumption good". In addition, country A can also arm and decide to invade country S. The extent of its armaments determines how much of the remaining endowment of the exhaustible resource of country S it can capture. After such a war, the government in country A follows a path of extraction of the remaining resources to maximize the utility of its citizens.

We study two alternative market structures. In the first, the entire stock of the exhaustible resource in country S is distributed among a set of perfectly competitive (price-taking) firms, which supply the world market. Country A consumers purchase at the world market price, unless there is a war (in which case, country A captures part of the endowment and the rest of the stock is destroyed). We refer to this as the *competitive environment* and look for a *Markov Perfect Competitive Equilibrium*, where all producers and consumers take future prices and probabilities of war as given, all markets clear, and country A's government maximizes the utility of the representative household.[1] In this competitive environment:

1. Country S producers do not internalize two types of externalities they create on each other. The first is the standard price effect, which is sometimes internalized by using "optimal" import and export taxes in international trade theory (for example, they can increase their revenues by reducing production and pushing international prices up, but do not do so because they are taking prices as given). The second externality is more interesting for our purposes and results from the military actions of country S in response to the equilibrium path of prices (the producers do not take into account that by changing their production plans they may be able to avoid war).

2. In any pure-strategy equilibrium, war cannot be delayed and it must necessarily occur in the initial period. If there were ever any war at some date $T > 0$, competitive producers would choose to extract all of their resource before that date, and it would therefore be beneficial for country A to declare war earlier, thus leading to the "unraveling of peace". This result is closely related to the fact that price-taking producers do not internalize the effects of their extraction decisions on the equilibrium probability of war, and suggests that in this environment war may be happening too soon and too frequently.

3. The incentives of country A to declare war depend on the elasticity of demand for the resource. In the empirically more relevant case where it is inelastic (elasticity less than one),[2] spending on the resource increases over time as its endowment is depleted. This

[1] The qualifier "Markov" is added because there is a parallel between the structure of equilibria here and in Markov Perfect Equilibria of dynamic political economy models, and the equilibrium will be in Markovian strategies. Moreover, when we consider the monopolistic environment, we will restrict attention to Markovian strategies (though there is no need to do so in the competitive environment).

[2] Several studies estimate the short-run demand elasticity for oil to be between 0.01 and 0.1, while the long-run elasticity is found to be higher but still less than 1 (see, for example, Gately and Huntington, 2002, Gately, 2004, or Cooper, 2003). The demand elasticity for other exhaustible resources may be greater, and throughout we provide results for any value the demand elasticity, though we place more emphasis on the inelastic case.

increases country A's incentives to declare war. In particular, when the military technology enables country A to capture a significant fraction of the remaining endowment (after incurring the cost of armament), spending will increase sufficiently at some point that war will be inevitable. But then the anticipation of future war encourages more rapid extraction and induces earlier war (in fact, when the elasticity of demand is constant, war will be in the initial date).

4. Despite the inefficiencies and the possibility of war, the path of extraction satisfies the Hotelling rule. Since firms are price-taking, equilibrium prices before war increase at the rate of interest as in the Hotelling rule, and optimal extraction after war by country A also satisfies the Hotelling rule.

The externalities in the competitive environment can be internalized if the government of country S regulates the price and the level of production of the resource (for example, by setting nonlinear taxes). We refer to the situation in which it does so as the *monopolistic environment*, because the government of country S is effectively acting as the monopoly supplier of the exhaustible resource. In fact, this is equivalent to a take-it-or-leave-it price-quantity offer to country A at each date, since the latter country always has the option of declaring war if it prefers this to trading at the price-quantity pair set by country S. Like the government of country A, we assume that the government of country S maximizes the net present discounted value of its citizens, and in doing so, it naturally takes into account the potential military threat from country A and how this threat will evolve over time. More formally, in this environment, we look for a *Markov Perfect Monopolistic Equilibrium*, where the two governments play best responses to each other in all subgames (in Markovian strategies). We show that in the monopolist environment:

1. Incentives to declare war in the monopolistic environment are again shaped by the elasticity of demand. When demand is inelastic, these incentives again increase over time as the resource is depleted.

2. Because with inelastic demand incentives to declare war and armaments increase as the resource is depleted, country S has an incentive to slow down the rate of extraction. As a consequence, extraction is slower and prices increase more slowly than that implied by the Hotelling rule.[3] Conversely, when demand is elastic, extraction is more rapid and prices increase more rapidly than the Hotelling benchmark.

3. Under certain conditions, in particular, when war is sufficiently costly for a country S, war is avoided in the monopolistic environment, even when it does take place under competitive

[3] We also show that if country A did not have an incentive to arm prior to war, then the Hotelling rule would always hold. This is because by standard efficiency arguments, country S would always make offers which avoid destructive war, and these offers would satisfy the Hotelling rule since this would provide both greater utility to country A and achieve greater revenues for country S.

markets. Moreover, in contrast to the competitive environment, war can be postponed and need not occur at the initial date even in pure-strategy equilibria.

4. A naive conjecture would be that the intervention by the country S government in the monopolistic environment would always prevent war and make country S households better off. This conjecture is not correct, however, even though the price externalities in the competitive environments are internalized. This is because there is a new source of inefficiency resulting from a commitment problem. In the competitive environment, country A needs to invest in armaments only when it intends to declare war. In the monopolistic environment, country S cannot commit to making an attractive offer to country A unless the latter arms. This implies that country A will have to make costly armament investments at each date in order to improve its terms of trade. But then, to prevent war, country S must pay for the future costs of armaments. This not only creates wasteful armament expenditures, but can make country S worse off and can lead to war even in situations in which there may not have been war in the competitive environment (though the opposite is also possible and in fact more likely as we have mentioned).

We also show that similar results obtain when country S can also invest in armaments for defense purposes, when there are multiple resource-for countries that compete for the resources of country S (and can go to war against each other to acquire these resources), and under more general assumptions on preferences.

We view this model as providing a first analysis of the interactions between dynamic equilibria (particularly the dynamics of resource extraction) and inter-country military action. The economic equilibrium is determined by the likelihood of war and the threat of war, and the path of prices affects the armaments and attack decisions of the resource-poor country. The framework formalizes the common concerns that the decreasing stocks of exhaustible resources may lead to a global war, and shows that because price-taking firms do not internalize the impact of their production decisions on other countries' military actions, war may happen too soon and too frequently. But our framework also suggests that these concerns may be exaggerated, particularly when resource-rich countries can regulate prices and production. Under these circumstances, the threat of war may not be realized and may simply affect the paths of prices and extraction of natural resources (in fact, precisely in such a way as to avoid war). Interestingly, in this scenario, intertemporal prices of natural resources no longer satisfy the Hotelling rule, and deviations from the Hotelling rule emerge as a way of intertemporally smoothing both the utility from the consumption of the exhaustible resource and the cost of armaments. However, our framework also shows that this type of regulation by the resource-rich countries leads to a new distortion as it induces the resource-poor country to increase its armaments to improve its terms of trade.

Whether the competitive or the monopolistic environment provides a better approximation to reality is an empirical question. Major oil producers such as Saudi Arabia and organizations such as OPEC regulate prices and quantities in the oil market, though many aspects of production both in the oil market and in other resource markets are still highly decentralized. Even though both the likelihood of war and the efficiency of the allocation of resources may be greater in one or the other environment, it is interesting that armament incentives and the path of prices are shaped by the elasticity of demand in both.

We should emphasize at this point that our analysis and thus our conclusions abstract from one important source of conflict, which undoubtedly intersects with scarcity of resources in practice: civil wars. Civil wars are fought over the control of increasingly valuable natural resources. The postwar history of much of sub-Saharan Africa has been marred by civil wars fought over control of oil and diamonds, and increasing prices of various natural resources might spur further internal conflicts in countries with weak institutions (e.g., Reno, 1998).[4] Investigating this issue as well as other effects of military conflict on trade patterns and on the terms of trade are interesting areas for future research.

Despite the importance of international conflict for economic and social outcomes and the often-hypothesized links between natural resources and international conflict, there are only a handful of papers discussing these issues. More specifically, our work contributes to the political economy of trade literature (e.g., Grossman and Helpman, 1995, Bagwell and Staiger, 1990, 2001, Maggi, 1999, Maggi and Rodriguez-Clare, 2007). Within this literature, our paper is most closely related to Antrás and Padró i Miquel (2009), who study how a dominant country can affect domestic politics in its trading partner. They show how lobbying type activities by the dominant country can be used for affecting policies and the terms of trade. In contrast to this literature, we emphasize how the ability to arm and to fight wars over resources affects patterns of trade. In this regard, our work is related to the war literature which explores how countries bargain with each other in order to avoid wars (e.g., Powell, 1999, Schwarz and Sonin, 2004, Skaperdas, 1992, and Yared, 2010). In contrast to this work, we study the two-way interaction between dynamic equilibria and the threat of war.[5] Finally, in our focus on the intertemporal allocation of exhaustible resources, our paper is related to the original seminal work of Hotelling (1931) together with important extensions by Dasgupta and Heal (1979).[6]

The paper is organized as follows. Section 3 describes the competitive environment and Section 4 describes the monopolistic environment. Section 5 considers extensions. Section 6 concludes and the Appendix includes additional proofs not included in the text.

[4]See, in particular, the comprehensive survey by Blatman and Miguel (2010). Recent research by Mehlum, Moene, and Torvik (2008) and Ross (2004) show that the negative effects of resources are confined to countries with weak institutions. Therefore, increasing threat of civil wars as natural resources become more scarce might also be confined to countries with such weak institutions.

[5]For related work on bargaining in the shadow of conflict, see also Acemoglu and Robinson (2006), Baliga and Sjöström (2004), Caselli (2005), Chassang and Padró i Miquel (2010), Dixit (1987), Fearon (1995), Garfinkel, Skaperdas, and Syropoulos (2009), Hirshleifer (1995), and Jackson and Morelli (2009).

[6]For other examples see also Kremer and Morcom (2000) and Pindyck (1979).

2 Environment

Time is discrete and denoted by $t = 0, ..., \infty$. There are two countries, A and S, and two goods, an exhaustible resource, to which we sometimes refer as "oil", and a perishable (non-resource) consumption good. Each country is inhabited by a continuum of mass 1 of identical households (or alternatively, by representative household). We assume that the governments in both countries maximize the intertemporal utility of their citizens (of the representative household in their country). In view of this, refer to actions by governments and countries interchangeably.

Households in country A receive the following flow utility from their consumption of the resource and the consumption good:

$$u\left(x_t^A\right) + c_t^A, \tag{1}$$

where $x_t^A \geq 0$ corresponds to their consumption of the resource and $c_t^A \gtreqless 0$ refers to the consumption good. The utility function $u(\cdot)$ is strictly increasing and a concave, i.e., $u'(\cdot) > 0$ and $u''(\cdot) < 0$, and satisfies the following Inada conditions $\lim_{x\to 0} u'(x) = \infty$ and $\lim_{x\to\infty} u'(x) = 0$. For simplicity, we assume that households in country S do not value the resource, and thus their utility is derived only from the consumption good:

$$c_t^S, \tag{2}$$

where $c_t^S \gtreqless 0$ refers to the consumption good. Households in both countries have a common discount factor $\beta \in (0, 1)$.

In each period both countries are endowed with an exogenous perishable amount of the consumption good. We normalize the endowment of this good for each country to zero (recall that negative consumption is allowed). In addition, country S is endowed with $e_0 > 0$ units of the exhaustible resource (oil) in period 0. To be consumed, the resource needs to be extracted, and we assume that extraction is at zero cost (and the amount extracted is non-storable and has to be consumed in the same period). We denote by x_t^S the amount of extraction of the resource in period t. The remaining stock of the non-extracted resource in period t, e_t, follows the law of motion

$$e_t = \sum_{k=0}^{\infty} x_{t+k}^S. \tag{3}$$

Country S extracts the resource and trades it for the consumption good with country A. We consider several trade environments in Sections 3 and 4.

In addition to trading, we allow country A to make two additional decisions in each period: how much to arm and whether to declare war against county S. The armament technology works as follows. At every date t, country A can choose a level of armament $m_t \in [0, \bar{m}]$ which has a per capita cost of $l(m_t)$ units of the consumption good. We assume that $l(\cdot)$ satisfies $l'(\cdot) > 0, l''(\cdot) \geq 0$, and $l(0) = 0$. The payoff from war depends on the amount of armament. If country A has armament m_t and attacks country S that has e_t units of the resource, country A obtains fraction $w(m_t)$ of e_t, while the remaining fraction $1 - w(m_t)$ is destroyed.[7] We assume that $w(\cdot)$ satisfies $w'(\cdot) \geq 0$, $w''(\cdot) \leq 0$, $w(m) \in [0,1]$ for all m with $\lim_{m \to \bar{m}} w'(m) = 0$, which imposes sufficient diminishing returns to armaments to ensure an interior level of equilibrium armaments. In most of the analysis, we allow for $\bar{m} = \infty$, in which case, $m_t \in [0, \infty)$ and $\lim_{m \to \infty} w'(m) = 0$. We use an indicator variable $f_T = 0$ to denote that no war occurred in periods $t = 0, .., T$ and $f_T = 1$ to denote that war in some period $t \leq T$.

If country A, after choosing m_t units of armament, attacks country S and the remaining endowment is e_t, the payoff to country A is $V(w(m_t)e_t) - l(m_t)$, where $l(m_t)$ is the cost of armament, incurred by the representative household in terms of the consumption good, and $V(w(m_t)e_t)$ is the continuation value of the representative household in that country starting with the ownership of the resource endowment of $w(m_t)e_t$ (since after war, the ownership of a fraction $w(m_t)$ of the remaining resource is transferred to the country A government). Since the government will use this stock to maximize the utility of its citizens, we have

$$V(w(m_t)e_t) = \max_{\{x_{t+k}, e_{t+k+1}\}_{k=0}^{\infty}} \sum_{k=0}^{\infty} \beta^k u(x_{t+k}) \tag{4}$$

subject to the resource and nonnegativity constraints, i.e.,

$$e_{t+1+k} = e_{t+k} - x_{t+k} \text{ for } k > 0, \tag{5}$$

$$e_{t+1} = w(m_t)e_t - x_t, \text{ and} \tag{6}$$

$$x_{t+k}, e_{t+k} \geq 0 \text{ for } k \geq 0. \tag{7}$$

In the event of a war, the payoff to country S is given by $\psi < 0$, which we think of as a large number, and in the limit, $\psi = -\infty$, meaning that the war is extremely costly to country S.

For future reference, it is useful to define $m^*(e)$ as the optimal amount of armament for country A if it attacks country S when country S has e units of resource endowment. Namely:

$$m^*(e) \equiv \arg\max_{m \geq 0} V(w(m)e) - l(m). \tag{8}$$

Given our assumptions on $u(\cdot)$, $w(\cdot)$, and $l(\cdot)$ (in particular, the Inada conditions), it is straightforward to see that $m^*(e) > 0$ and is a continuously differentiable function of e for all $e > 0$.

[7] To facilitate interpretation, we model the outcome of war as deterministic—in particular, with country A grabbing a fixed fraction of the resource. This is largely without loss of any generality. All of our results apply to an environment in which the outcome of war is stochastic, provided that after war, the two countries never interact again. For example, we can have $w(m_t)$ as the probability that country A receives a fraction λ^H of the endowment and $1 - w(m_t)$ as the probability that it receives a fraction $\lambda^L < \lambda^H$ of the endowment.

3 Competitive Environment

We start by considering a competitive environment in which trade occurs at market clearing prices and both buyers and sellers take these prices as given. This environment will allow us to highlight the key economic forces that determine incentives to fight and to illustrate the externalities in the competitive environment.

3.1 Markov Perfect Competitive Equilibrium

In the competitive environment, country S has a unit measure of firms. Each firm is labeled by i and owns an equal fraction of the total natural resource endowment of country S. Firm i extracts resources x_{it}^S and sells them in a competitive market at price p_t in units of consumption good. All profits are rebated to households of country S as dividends. We will next define a notion of "competitive equilibrium" for this environment. This definition requires some care, since producers in country S are price takers, but must also recognize the likelihood of war, which result from the strategic choices of the government of country A. We define the notion of equilibrium in two steps. First, we impose price taking and market clearing for all relevant Arrow-Debreu commodities, i.e., for the resource at each date following any history (by Walras law, this guarantees market clearing for the consumption good). Second, we study the problem of country A taking the relationship between the probability of war and these prices as given.

Price-taking implies that each firm i in country S chooses extraction plan $\{x_{it}^S\}_{t=0}^{\infty}$ to maximize its expected profits at time $t = 0$ (according to its information set then)

$$\max_{\{x_{it}^S\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t p_t x_{it}^S \tag{9}$$

subject to the constraints

$$e_{it+1} = e_{it} - x_{it}^S \text{ if } f_t = 0$$

$$e_{it+1} = 0 \text{ if } f_t = 1, \text{ and}$$

$$x_{it}^S, e_{it+1} \geq 0 \text{ for all } t \geq 0.$$

The second constraint stems from the fact that firm i loses its endowment if country A declares war. The solution to this problem implies that

$$x_{it}^S \in \begin{cases} 0 & \text{if } p_t < \beta p_{t+1} \Pr\{f_{t+1} = 0\} \\ [0, e_{it}] & \text{if } p_t = \beta p_{t+1} \Pr\{f_{t+1} = 0\} \\ e_{it} & \text{if } p_t > \beta p_{t+1} \Pr\{f_{t+1} = 0\} \end{cases}. \tag{10}$$

Equation (10) captures the fact that firms do not only take into account future prices, but also the future probability of war in deciding how much to extract today.

Similarly, the representative household in country A chooses the demand for resource x_t^A as a solution to

$$\max_{x_t \geq 0} u(x_t^A) - p_t x_t^A, \tag{11}$$

which gives us the standard optimality condition

$$u'(x_t^A) = p_t. \tag{12}$$

We denote the total supply of the resource by x_t^S. Market clearing implies that the price sequence $\{p_t\}_{t=0}^{\infty}$ must be such that

$$x_t^S = x_t^A \tag{13}$$

for all t.

In addition, the country A government can impose a lump sum tax on its citizens of size $l(m_t)$ in order order to invest in armament m_t, and it can choose to attack country S at any date.

More specifically, we consider the following sequence of events. Since the game is trivial after the war has occurred, we only focus on the histories for which war has not occurred yet (i.e. $f_{t-1} = 0$).

1. Country A's government chooses a level of armament $m_t \geq 0$.
2. Firms in country S commit to extraction $x_t^S \geq 0$ and households in country A commit to consumption x_t^A at prices p_t in the event that country A does not attack country S at stage 3.
3. Country A's government decides whether or not to attack country S.
4. Extraction and consumption take place.

Note that in stage 2, firms and households trade contingent claims on the resource, where the contingency regards whether or not war is declared at stage 3.[8]

We can now define a *Markov Perfect Competitive Equilibrium* (MPCE) formally. Given that the game is trivial after the war has occurred, we only define strategies for dates t for which $f_{t-1} = 0$. Denote the strategy of the government of country A as φ which consists of a pair of functions φ^m and φ^f. In each period, the function φ^m assigns a probability distribution over armament decisions m_t as a function of e_t. The function φ^f assigns a probability distribution with which country S attacks as a function of $\left(e_t, m_t, p_t, x_t^S, x_t^A\right)$.

Firms and households take price sequences and the sequence of policies by the government of country A as given. It is important to note that even if war is expected with probability 1 at

[8]We could alternatively simplify the timing of the game by allowing country A to arm and to make its attacking decisions in the first stage, and then, if the attack did not occur, households and firms would trade in the second stage. Under our notion of equilibrium, these two set ups are equivalent. We chose this set up to be consistent with the timing of the game in Section 4.

date t, their choices do take into account the continuation strategy of the government and the future sequence of prices from $t+1$ onward in the event that war is not actually declared at t. Therefore, allocations and prices conditional on war never being declared need to be specified as part of the equilibrium. To do this, let us define a sequence $\gamma \equiv \left\{e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right\}_{t=0}^{\infty}$, where each element at t corresponds to the values of $\left(e_t, p_t, x_t^S, x_t^A\right)$ which would emerge if $f_{t-1}=0$. Given such a sequence γ, one can define $U_A\left(e_t^*\right)$ as the welfare of (the representative household in) country starting from e_t^* conditional on $f_{t-1}=0$. Given this definition, the period t payoff to country A, starting from e_t^* under $f_{t-1}=0$ and conditional on some choice (m_t, f_t), is

$$\left(1-f_t\right)\left(u\left(x_t^{A*}\right)-p_t^* x_t^{A*}+\beta U_A\left(e_{t+1}^*\right)\right)+f_t V\left(w\left(m_t\right) e_t^*\right)-l\left(m_t\right). \tag{14}$$

The first term is the value in case of no war, while the second term is the continuation value following war.

Definition 1 *A Markov Perfect Competitive Equilibrium (MPCE) consists of φ and γ such that:*

1. *Given γ, φ^m maximizes* (14) *for every e_t^* in γ.*
2. *φ^f maximizes* (14) *given m_t for every $\left(e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right)$ in γ, and*
3. *γ satisfies* (3), (10), (12), *and* (13) *with* $\Pr\left\{f_{t+1}=0\right\}=\varphi^f\left(e_{t+1}^*, m_{t+1}^*, p_{t+1}^*, x_{t+1}^{S*}, x_{t+1}^{A*}\right)$.

Under this definition of equilibrium, the government in country A makes its armament and fighting decisions optimally today, taking into account its future behavior and that of the private sector in the event that war is not declared today.[9] Furthermore, firms and households behave optimally today, taking into account the future behavior of the government in the event that war is not declared today. In our definition, we impose that the continuation equilibrium in the event that war does not happen today must always be such that households and firms optimize, markets clear, and country A is choosing its best response.

Without imposing further restrictions, there is an indeterminacy of equilibria at the point where $e_t=0$ because country A would then be indifferent between choosing $f_t=0$ on the one hand and $m_t=0$ and $f_t=1$ on the other (provided that $u(0)$ is finite). To avoid this economically uninteresting multiplicity, throughout we suppose that there is a cost of war equal to $\upsilon>0$ for country A and consider equilibria in the limiting case $\upsilon \rightarrow 0$. Throughout, MPCE refers to such limiting equilibria or "refined" MPCE.[10] Consequently, when $u(0)$ is finite, MPCE will involve no war at $e_t=0$. When $u(0)=-\infty$, there may still be war at $e_t=0$, and we formally analyze this limit in the Appendix.

[9] We have put the qualifier *Markov* for the reasons outlined in footnote 1.

[10] This refinement is in the spirit of "trembling hand perfection" and rules out equilibria supported by weakly dominated strategies for country A.

3.2 Analysis

Our first result establishes the existence of (refined) MPCE in the above-described environment.

Lemma 1 *An MPCE exists.*

Proof. See Appendix. ■

We next characterize MPCEs. As a benchmark, it is useful to consider a case when country A cannot arm and attack by restricting $f_t = 0$ for all t. In that case there is no uncertainty and the first-order conditions to (10), (12), and (13) imply that the equilibrium prices p_t must satisfy

$$\beta p_{t+1} = p_t. \tag{15}$$

This is a market form of the famous Hotelling rule and requires that prices of the exhaustible resource grow at the rate of interests, which is also equal to the discount rate $((1-\beta)/\beta)$. The intuition is straightforward: since producers are price-taking and can extract the resource at no cost, there will only be positive extraction at all dates if they make the same profits by extracting at any date, which implies (15). Moreover, given the Inada conditions on the utility function and (12), having zero extraction at some date is not consistent with equilibrium. Hence (15) must hold in any MPCE.

The connection between (15) and be Hotelling rule can be seen more explicitly by using (10), (12), and (13), which imply that the sequence of resource consumption $\{x_t\}_{t=0}^{\infty}$ must satisfy

$$\beta u'(x_{t+1}) = u'(x_t) \tag{16}$$

at all t, which is the familiar form of the Hotelling rule (with zero extraction costs).

We next turn to country A's armament and war decisions and characterize MPCE. We first consider pure-strategy equilibria (where φ^f is either 0 or 1 at each date). This gives us our first result, showing that because of the externalities that the production decisions of price-taking firms create on others, wars cannot be delayed in pure-strategy equilibria.

Proposition 1 *In any pure-strategy MPCE:*

1. *War can only occur at $t = 0$, and*

2. *The equilibrium sequence of resource extraction, x_t, satisfies (16) for all t.*

Proof. Suppose country A attacks in period period $T > 0$ with probability 1. From (10), firms extract all the resource before period T, so that $e_t = 0$ for some $t \leq T$. This implies that $x_T = 0$. First, suppose $u(0)$ is finite. In this case country A does not attack country S over zero endowment, no attack occurs in period T leading to a contradiction. Alternatively, suppose that $u(0) = -\infty$. In this case the equilibrium payoff for country A in period 0 is $-\infty$. If country

A deviates in period 0 and chooses the level of armament $m^*(e_0)$ and attacks country S, its equilibrium pay-off is

$$V(w(m^*(e_0))e_0) - l(m^*(e_0)) > -\infty.$$

Therefore country A attacks country S in period 0 and no attack occurs in period T. This establishes the first part of the proposition. If an attack occurs in period 0, the first-order conditions to (4) imply that x_t must satisfy (16). If no attack occurs in period 0, $f_t = 0$ for all t and the argument preceding the proposition establishes (16). ■

This proposition shows that in pure-strategy equilibria, wars cannot be delayed. The intuition is simple and directly related to the externalities across firms: if there is a war at time T, price-taking firms will deplete their entire endowment before T, and this will encourage war to be declared earlier.

This result also implies that along the equilibrium path, consumption of the resource satisfies the Hotelling rule (16) and that there are no intertemporal distortions. If there is no war at $t = 0$, then the equilibrium is identical to the benchmark competitive equilibrium in which war is not possible. If there is war at $t = 0$, then country A seizes a fraction $w\left(m^*\left(e_0\right)\right)$ of the initial endowment and it extracts resources according to (16) since this maximizes the welfare of households in country A.

To further characterize under which conditions wars may occur and to explore the possibility of mixed strategy equilibria, we restrict attention to utility functions that imply a constant elasticity of demand for the resource. This is the same as the commonly used class of constant relative risk aversion (CRRA) or iso-elastic preferences:

$$u(x) = \frac{x^{1-1/\sigma} - 1}{1 - 1/\sigma} \tag{17}$$

for $\sigma > 0$. Clearly, the elasticity of demand for the exhaustible resource is constant and equal to $-u'\left(x\right)/\left(xu''\left(x\right)\right) = \sigma$. As we will see, when $\sigma < 1$, which is the empirically relevant case for oil (and perhaps also for other exhaustible resources), total spending on the exhaustible resource increases over time as its endowment is depleted and the price increase dominates the reduction in quantity. When preferences take this form, we can generalize Proposition 1 to any MPCE (i.e., also those in mixed strategies) provided that $\sigma \neq 1$.

Proposition 2 *Suppose preferences satisfy* (17) *and* $\sigma \neq 1$. *Then in any mixed-strategy MPCE:*

1. *War can only occur at* $t = 0$, *and*

2. *The equilibrium sequence of resource extraction,* x_t, *satisfies (16) for all* t.

Proof. See Appendix. ■

To understand the intuition for this proposition it is useful to consider how country A's incentives to declare war change over time as the endowment of the exhaustible resource declines.

To do this, consider the special case where $w(\cdot)$ is a step function. In particular, if country A invests $\tilde{m} > 0$ in armament, it will receive the entire remaining endowment of the exhaustible resource, i.e., $w(\tilde{m}) = 1$. If it invests less, it will obtain none of the endowment. This functional form implies that country A is effectively choosing between zero armaments (and no war), and armaments equal to $\tilde{m}$ to obtain the entire endowment of the resource. Suppose further that country A is choosing between going to war at time t and permanent peace thereafter. Thus if it does not declare war at time t, the subsequent allocations are given by the standard competitive equilibrium allocations, denoted by $\left\{x_{t+k}^{ce}, p_{t+k}^{ce}, e_{t+k}^{ce}\right\}_{k=0}^{\infty}$. It is straightforward to show that $x_{t+k}^{ce} = (1-\beta^{\sigma})\, e_{t+k}^{ce}$, $p_{t+k}^{ce} = \left(x_{t+k}^{ce}\right)^{-1/\sigma}$, and $e_{t+k+1}^{ce} = e_{t+k}^{ce} - x_{t+k}^{ce}$. This implies that the payoff to country A in period t from not going to war is equal to

$$
\begin{aligned}
U^{ce}(e_t) &= \sum_{k=0}^{\infty} \beta^k u(x_{t+k}^{ce}) - \sum_{k=0}^{\infty} \beta^k p_{t+k}^{ce} x_{t+k}^{ce} \\
&= \sum_{k=0}^{\infty} \beta^k u(x_{t+k}^{ce}) - (1-\beta^{\sigma})^{-1/\sigma} \left(e_t^{ce}\right)^{1-1/\sigma}.
\end{aligned}
$$

If country A invests $\tilde{m}$ in armament in period t and declares war, then, under the assumption here that $w(\tilde{m}) = 1$, its payoff is given by

$$
V(w(\tilde{m})e_t) - l(\tilde{m}) = \sum_{k=0}^{\infty} \beta^k u(x_{t+k}^{ce}) - l(\tilde{m}).
$$

This implies that the difference between the payoffs from war and no war is equal to

$$
V(w(\tilde{m})e_t) - l(\tilde{m}) - U^{ce}(e_t) = (1-\beta^{\sigma})^{-1/\sigma} \left(e_t^{ce}\right)^{1-1/\sigma} - l(\tilde{m}).
$$

Since $\{e_t^{ce}\}_{t=0}^{\infty}$ is a decreasing sequence by construction, this expression monotonically decreases to zero if σ is greater than 1 and monotonically increases towards infinity if σ is less than 1. Therefore, depending on the elasticity of demand for the resource, the payoff from war either monotonically converges to zero or becomes unboundedly large. Which of these two cases applies depends on whether the payments that country A makes to country S in competitive equilibrium, $\sum_{k=0}^{\infty} \beta^k p_{t+k}^{ce} x_{t+k}^{ce}$, converge to zero or infinity. This logic allows us to show in the proof of Proposition 2 that if demand is elastic (σ is greater than one), incentives to fight must be decreasing for country A. In particular, if it weakly prefers peace to war in any period t, it strictly prefers peace in all the subsequent periods. Alternatively, if the demand for the resource is inelastic (σ is less than one), incentives to fight must be increasing and country A eventually prefers war in which case the arguments of Proposition 1 apply directly. In particular in this case war must occur with probability 1 independently of the cost of armaments $l(\tilde{m})$ and the cost of war to country S. It is straightforward to see that the same conclusion holds if country A could, as in our model, choose to go to war at any date it wishes.

This special case illustrates the key intuition underlying Proposition 2. More generally, war has an additional cost for country A, which is that a fraction $1 - w(m^*(e))$ of the endowment is lost in war. If this cost is sufficiently high, country A may prefer not to attack country S even if its equilibrium payments $\sum_{k=0}^{\infty} \beta^k p_{t+k} x_{t+k}$ diverge to infinity. All the same, the main insights and the factors affecting the comparison between war and no war remain the same as in the case where $w(\tilde{m}) = 1$.

The next proposition contains the main result of the competitive environment. It characterizes the conditions under which a pure-strategy equilibrium exists and when it will involve war.

Proposition 3 *Suppose $l(\cdot)$ and $w(\cdot)$ satisfy the assumptions in Section 2. Then:*

1. *Suppose $\sigma > 1$. Then there exists $\widehat{e} > 0$ such that if $e_0 < \widehat{e}$, then the unique MPCE is in pure strategies and has no war along the equilibrium path, and if $e_0 > \widehat{e}$, then the unique MPCE is in mixed strategies and has war at date 0 with probability 1.*

2. *Suppose $\sigma < 1$. Then there exists $\widehat{w} < 1$ such that if $\lim_{m \to \bar{m}} w(m) < \widehat{w}$, the unique MPCE is in pure strategies and has no war along the equilibrium path, and if $\lim_{m \to \bar{m}} w(m) > \widehat{w}$, the unique MPCE is in pure strategies and has war at date 0 with probability 1.*

Proof. See Appendix. ■

This proposition therefore shows that in the empirically more relevant case where $\sigma < 1$, the equilibrium is always in pure strategies, and moreover, provided that country A is capable of capturing most of the remaining endowment of the resource, the equilibrium will involve war at the initial date. The intuition for this result follows from Proposition 2. When $\sigma < 1$, spending on the resource and incentives to declare war increase over time. If, by spending the necessary resources, country A can capture a sufficient fraction of the remaining endowment of the resource, it will necessarily find it optimal to declare war at some point. The condition $\lim_{m \to \bar{m}} w(m) > \widehat{w}$ ensures the latter requirement. But we know from Proposition 1 that if war will occur in pure strategies, it must occur at the initial date. In particular, anticipating war, country S produces would always deplete the entire resource and this induces country A to jump the gun and declare war at the initial date. Notably, this conclusion is independent of the cost of war to country A, i.e., the function $l(\cdot)$, and the cost of war to country S, $-\psi$. In particular, this proposition applies even if $\psi = -\infty$. In this case, of course, war is extremely costly to the citizens of country S, but under our assumption that resource extraction takes place competitively, firms in this country can take no action to stave off a very costly war. This is one of the main motivations for our analysis of the "monopolistic" environment, where such actions will be possible. For future reference, we state this simple implication of Proposition 3 as a corollary.

Corollary 1 *If $\sigma < 1$ and if $\lim_{m \to \bar{m}} w(m)$ is sufficiently close to 1, then war will take place at date 0 even if $\psi = -\infty$.*

Finally, Proposition 3 does not cover the knife-edge case where $\sigma = 1$, which turns out to be more complicated. When $\sigma = 1$, the demand for the resource has unitary elasticity and the equilibrium payment $p_t x_t$ is constant over time (independent of e_t). It can then be shown that when $\sigma = 1$ and when there exists a pure-strategy equilibrium with no war, there also exist mixed-strategy equilibria. In particular, country A might mix with a constant probability between war and no war at each date. When such a mixed-strategy equilibrium exists, it will involve equilibrium prices that rise at a faster rate than $(1-\beta)/\beta$ and equilibrium allocations and prices will deviate from the Hotelling rule (16). Since such equilibria are only possible in the knife-edge case where $\sigma = 1$, we do not dwell on them. Hence, we can also conclude that in the competitive environment, the extraction patterns and prices always satisfy the Hotelling rule. We will see that this is also not true in the monopolistic environment we study next.

4 Monopolistic Environment

From the point of view of country S, the competitive equilibrium is suboptimal for two reasons. The first is the standard price effect. Each producer, by extracting more, is reducing the price faced by other producers. In traditional trade models, this price effect is sometimes internalized by using "optimal" import and export taxes. The second is a novel externality resulting from the military actions of country A in response to the equilibrium path of prices. Recall the second part of Proposition 3 with $w(\tilde{m}) = 1$ and $\sigma < 1$. In this case, war is unavoidable under competitive markets and occurs immediately, even though the cost of war, $-\psi$, may be arbitrarily high for country S. War occurs because as the price of the resource increases, payments from country A households to country S firms become arbitrarily large. Yet price-taking firms do not internalize that high resource prices increase incentives to fight for country A. If country S could somehow reduce these payments, it may be able to avoid war. The government of country S might, for example, impose regulations on the price of the resource in order to prevent war or to improve the welfare of its citizens. In this section we study equilibrium allocations under such regulation. We will see that by regulating the levels of prices and production, the government of country S can indeed internalize the externalities, and that a consequence of this will be deviations of prices from the Hotelling rule. However, we will also see that this type of monopolistic behavior by country S introduces a new externality due to its inability to commit to providing attractive terms of trade to country A. Consequently, we will see that even though the monopolistic environment may be more effective at preventing war under certain conditions, it can also increase the likelihood of war and may even make country S worse off, despite its ability to act as the monopolist (Stackleberg leader) in its interactions with country A.

4.1 Timing of Events and Markov Perfect Monopolistic Equilibrium

We consider the simplest possible way of modeling such regulations, which will involve the country S government acting as a "monopolist" that sets prices and quantities recognizing their implications for current and future economic and military actions. In particular, suppose that the government sets nonlinear tariffs to control both the level of the price of the resource and its production. Given this resulting price-quantity pair, country A can still declare war. This environment is evidently equivalent to one in which country S makes a take-it-or-leave-it price-quantity offer to country A. In what follows, we directly study a game in which country S makes such offers (and do not explicitly introduce the nonlinear tariffs to save on notation).

More specifically, we consider the following game. At every date t at which war has not yet occurred, country A chooses the level of armament m_t. Next, (the government of) country S makes a take-it-or-leave-it offer $z_t = \{x_t^o, c_t^o\}$ to country A, consisting of an offered delivery of x_t^o units of the resource in exchange for $-c_t^o$ units of the consumption good. Country A then accepts or rejects this offer, which is denoted by $a_t = \{0, 1\}$, with $a_t = 1$ corresponding to acceptance. Conditional on rejecting the offer, country A then chooses whether or not to declare war on country S. As in Section 3, the continuation payoff to country A following war is $V(w(m_t) e_t) - l(m_t)$, and the continuation payoff for country S is ψ. If country A accepts the offer, then the the flow utilities to households in country A and S are $u(x_t^o) + c_t^o - l(m_t)$ and $-c_t^o$, respectively. If instead country A rejects the offer and does not declare war, then the flow utilities to households in country A and S are $u(0) - l(m_t)$ and 0, respectively

We formally summarize the order of events for all periods t for which $f_{t-1} = 0$ as follows:

1. Country A's government chooses a level of armament m_t.
2. Country S's government makes a take-it-or-leave-it offer z_t to country A.
3. Country A's government decides whether or not to accept the offer a_t. If $a_t = 0$, it can declare war by choosing f_t.
4. Extraction and consumption take place.

The timing of events makes it clear that this is a dynamic game between the two countries, and we consider its Markov Perfect Equilibrium, which refer to as *Markov Perfect Monopolistic Equilibrium* (MPME). This equilibrium is similar to an MPCE with the exception that firm and consumer optimality is no longer required, since country S's and country A's governments jointly determine the transfer of goods across countries. In such an equilibrium all actions depend only on payoff relevant state variables, which here include the endowment, e_t, and prior actions at the same date. As we did in the analysis of MPCE, we define strategies for dates t in which $f_{t-1} = 0$ (i.e., for histories where war has not yet occurred).[11]

Let country A's strategy be presented by $\phi_A = \left\{\phi_A^m, \phi_A^a, \phi_A^f\right\}$. Here ϕ_A^m assigns an armament decision for every e_t; ϕ_A^a assigns an acceptance decision for every (e_t, m_t, x_t^o, c_t^o); and ϕ_A^f assigns a

[11] In this case, as opposed to the MPCE, the restriction to Markovian strategies is significant. There also exist non-Markovian equilibria, where country A can threaten war in the future without arming today. If the discount factor, β, is sufficiently less than one, then we expect that qualitative behavior in these non-Markovian equilibria to be similar to MPME that we characterize here as the threat of future war will have limited impact on current behavior.

war decision for every $(e_t, m_t, x_t^o, c_t^o, a_t)$, where this decision is constrained to 0 if $a_t = 1$. Country S's strategy is denoted by ϕ_S and consists of an offer for every (e_t, m_t). We allow mixed strategies for both countries though it will become clear later that only pure strategies are relevant for all, except for knife-edge, cases. We next provide a formal definition of equilibrium.

Definition 2 *A Markov Perfect Monopolistic Equilibrium (MPME) is a pair* $\{\phi_A, \phi_S\}$ *where*

1. *Given* ϕ_S, ϕ_A^m *maximizes the welfare of country A for every* e_t, ϕ_A^a *maximizes the welfare of country A for every* (e_t, m_t, x_t^o, c_t^o), *and* ϕ_A^f *maximizes the welfare of country A for every* $(e_t, m_t, x_t^o, c_t^o, a_t)$ *subject to* $f_t = 0$ *if* $a_t = 1$.

2. *Given* ϕ_A, ϕ_S *maximizes the welfare of country S for every* (e_t, m_t) *subject to (3).*

Given these strategies, we define the equilibrium continuation values $\{U_A(e_t), U_S(e_t)\}$ to countries A and S which constitute the continuation value to each country at the beginning of the stage game at t conditional on no war in the past. Similar to equation (14) in the previous section, these continuation values are given by

$$\begin{aligned} U_A(e_t) &= (1 - f_t)(u(x_t) + c_t + \beta U_A(e_{t+1})) + f_t(V(w(m_t)e_t) - l(m_t)) \\ U_S(e_t) &= (1 - f_t)(-c_t + \beta U_S(e_{t+1})) + f_t\psi. \end{aligned}$$

4.2 Analysis

We next characterize the MPME. We show that unlike the competitive environment, the time path of resource extraction is distorted away from the Hotelling rule.[12] Despite this difference in price paths, many qualitative features of equilibrium are shaped by the same forces as in the competitive environment, in particular, by whether the elasticity of demand is greater than or less than one, which determines whether incentives to declare war increase or decrease over time. We also show that country S may delay wars or avoid them entirely in some of the cases when wars are unavoidable under competitive markets. Nevertheless, a naive conjecture that the monopolistic environment will necessarily reduce the likelihood of war and will make country S, which is now acting as a Stackleberg leader and making take-it-or-leave-it offers, better off is not correct. In fact, it is possible for war to occur in a monopolistic equilibrium in cases when war can be avoided under competitive markets, and country S can have lower utility. Both of these are because of a new source of distortion in the monopolistic environment, resulting from the fact that country S cannot commit to making attractive price-quantity offers to country A; this, in turn, induces country A to invest in armaments at each date in order to improve its terms of trade.

[12] The key reason for distortions in the monopolistic equilibrium is the armament decision of country A. To highlight how armament affects the distortion, in the Appendix we analyze the case where country A can attack country S without arming. We show that in this case wars never occur and the path of resource extraction satisfies the Hotelling rule (16).

We first consider the optimal strategy for country S for a given level of armament m_t. Let $\tilde{U}_S(e_t; m_t)$ be the value function of country S when its price-quantity offer is accepted by country A, starting with endowment e_t and armament level of country A equal to m_t. This value function is given by the following recursion

$$\tilde{U}_S(e_t; m_t) = \max_{x_t \geq 0, c_t} \{-c_t + \beta U_S(e_{t+1})\} \tag{18}$$

subject to the resource constraint (3), and the participation constraint of country A, given by

$$u(x_t) + c_t - l(m_t) + \beta U_A(e_{t+1}) \geq V(w(m_t)e_t) - l(m_t). \tag{19}$$

Constraint (19) requires the value of country A when it accepts the price-quantity offer (x_t, c_t) at time t to be greater than its utility if it declares war and captures a fraction $w(m_t)$ of the remaining endowment of country S. In theory, this value also needs to be greater than the continuation value from rejecting the price-quantity offer but not declaring war. But it can be easily verified that this latter option is never attractive for country A, and hence there is no need to specify it as an additional constraint in the maximization problem (18).[13]

Moreover, it is straightforward to see that constraint (19) must bind in equilibrium, since otherwise country S could make an offer with sightly greater transfers and would increase its payoff. Finally, if $\tilde{U}_S(e_t; m_t)$ is less than the payoff from war ψ, the best response for country S is to make any offer that violates (19). Thus in equilibrium, starting from (e_t, m_t), the payoff of country A is equal to

$$V(w(m_t)e_t) - l(m_t) \tag{20}$$

regardless of whether it accepts the price-quantity offer of country S. This implies that country A's best response is always to choose a level of armament maximizing (20). We defined this level of armaments as $m^*(e_t)$ in equation (8). Therefore, the equilibrium payoffs for countries A and S can be written as:

$$U_A(e_t) = V(w(m^*(e_t))e_t) - l(m^*(e_t)) \tag{21}$$

and

$$U_S(e_t) = \max\left\{\tilde{U}_S(e_t; m^*(e_t)); \psi\right\}. \tag{22}$$

We next show that an MPME exists.

[13] In particular, this additional constraint can be written as

$$u(x_t) + c_t - l(m_t) + \beta U_A(e_{t+1}) \geq u(0) - l(m_t) + \beta U_A(e_t).$$

Suppose, to obtain a contradiction, that both this constraint and (19) bind. By definition, $U_A(e_t) = u(x_t) - l(m_t) + c_t + \beta U_A(e_{t+1})$. Combining this with (19), we obtain $U_A(e_t) = u(0)/(1-\beta)$, which is necessarily less than $V(w(m_t)e_t)$, showing that this constraint cannot be binding.

Lemma 2 *An MPME exists.*

Proof. See Appendix. ■

We now turn to the first main result of this section.

Proposition 4 *Suppose w satisfies assumptions of Section 2. In any MPME, if $f_{t+1} = 0$, then*

$$\begin{aligned} \beta u'(x_{t+1}) &> u'(x_t) \text{ if } m^{*\prime}(e_{t+1}) > 0, \text{ and} \\ \beta u'(x_{t+1}) &< u'(x_t) \text{ if } m^{*\prime}(e_{t+1}) < 0. \end{aligned} \tag{23}$$

Proof. See Appendix. ■

The main technical difficulty in the proof of this proposition lies in the fact that the value function $U_S(e_t)$ may not be differentiable and we use perturbation arguments in the Appendix to prove this result. It is easy to verify it heuristically if one assumes differentiability. To do this, let us substitute (21) into (18), taking into account that since $f_{t+1} = 0$, it is the case that $\tilde{U}_S(e_t; m_t) = U_S(e_t)$. Take the first-order conditions to obtain

$$u'(x_t) - \beta u'(x_{t+1}) + \beta l'(m^*(e_{t+1}))\, m^{*\prime}(e_{t+1}) = 0. \tag{24}$$

Since $l'(\cdot) > 0$, equation (24) implies (23).

Proposition 4 shows that the key determinant of the growth rate of the shadow price of the resource is whether country A increases or decreases armaments as the resource stock declines. The important underlying reason for this result comes from the inabilities of countries to commit to long term contracts. If country S could commit in period 0 to a sequence of offers $\{z_t\}_{t=0}^{\infty}$, only one time investments in the armaments of country A would be needed, no war would occur, and the shadow price of the resource would grow with a rate of time preference.

In this model this commitment is not possible. Country A needs to invest in armament in each period to obtain better terms of trade from country S. In particular, given the timing of events above, it is clear that country A will choose armaments at each date in order to maximize its continuation value $V(w(m(e_t))\, e_t)$, since this will be its utility given country S's take-it-or-leave-it offer. This continuation value incorporates the sequence of future armament costs as well, and so country S will take these into account also when deciding path of extraction and prices. To develop this intuition further, let us substitute (21) into (19):

$$u(x_t) + c_t + \beta\left(V(w(m^*(e_{t+1}))\, e_{t+1}) - l(m^*(e_{t+1}))\right) \geq V(w(m^*(e_t))\, e_t). \tag{25}$$

Suppose that armaments increase as the resource stock decreases. The increase in m_t implies that constraint (25) becomes harder to satisfy over time. If country S extracts ϵ units of resources less in period t and ϵ more in period $t+1$, holding everything fixed, it changes payoff of country A by $(\beta u'(x_{t+1}) - u'(x_t))\,\epsilon$. In addition, it relaxes constraint (25) since the stock of resources is higher so that armament by country A declines, and this allows country S to decrease the offer of c_t. Therefore, as long as $\beta u'(x_{t+1}) - u'(x_t) \geq 0$, country S can be made better off postponing resource extraction into the next period. Thus, it must be the case that $\beta u'(x_{t+1}) - u'(x_t) < 0$ in equilibrium. When the amount of armament is decreasing in e_t, this effect works in the opposite direction.

Note that Proposition 4 does not specify under what conditions on the primitives $m^*(e_t)$ is increasing or decreasing in e. This is stated in the next proposition.

Proposition 5 *If* $-u'(x) / (xu''(x)) < 1$ *for all* x*, then* $m^{*\prime}(e_t) < 0$*. Conversely, if* $-u'(x) / (xu''(x)) > 1$ *for all* x*, then* $m^{*\prime}(e_t) > 0$*.*

Proof. See the Appendix. ■

This proposition relates the sign of $m^{*\prime}(e_t)$ and the growth rate of the shadow value of the natural resource to the elasticity of demand for the resource. An immediate corollary that further clarifies the result in the proposition is presented next:

Corollary 2 *Suppose preferences are given by (17). Then if* $\sigma < 1$, $m^{*\prime}(e_t) < 0$*, and if* $\sigma > 1$, $m^{*\prime}(e_t) > 0$*.*

Proposition 5 and the corollary are intuitive. We saw in Section 3 that elasticity of demand played a crucial role in determining whether incentives to declare war increase or decrease as the endowment of the resource is depleted. The same effect determines the equilibrium armaments for country A in the monopolistic environment. When $\sigma < 1$, demand is inelastic and the value of the resource, $V'(e_t)e_t$, increases over time. This induces country A to invest more in armaments. Country S internalizes the effect of resource depletion on country A's incentives to arm (as it can hold country A down to its continuation value). It then counteracts the rise in country A's armament costs by reducing the rate of resource extraction. This is equivalent to a (shadow) price sequence growing at a slower rate than $(1 - \beta) / \beta$ (the Hotelling benchmark). In contrast, if $\sigma > 1$, demand is elastic and the value of the resource and country A's armaments are decreasing in the endowment. In this case, country S can further reduce country A's armament costs by raising the rate of resource depletion.

A natural question concerns the conditions under which war or occurs or war is avoided in the monopolistic environment. A naive conjecture is that country S's ability to regulate the price and the level of production of the resource will make wars less likely and its citizens better off relative to their level of utility in the competitive equilibrium. This conjecture is not correct, however, because there is also a new distortion in the monopolistic environment. Recall that at each date country S makes a price-quantity that gives to country A utility equal to $V(w(m_t)e_t) - l(m_t)$. It cannot commit to giving a higher utility to country A—unless the latter invests more in armaments. This implies that unless country A invests in armaments at each date, it will not receive favorable terms of trades. Therefore, the monopolistic environment encourages investments in armaments at each date, whereas in the competitive environment country A did not need to invest in arms in periods in which it did not declare war. Moreover, since country S needs to give country A at least utility $U_A(e_t) = V(w(m^*(e_t))e_t) - l(m^*(e_t))$, it effectively pays for country A's future costs of armaments, so country S may be made worse off by its ability to make take-it-or-leave-it offers, or by its inability to make a commitment to future paths of prices and production. The next proposition exploits this new distortion and shows why the above-mentioned conjecture is incorrect.

Proposition 6 *In an MPME,*

1. *War is avoided if preferences satisfy* (17) *for* $\sigma < 1$ *and*

$$-\beta l(\overline{m}) > \psi(1-\beta), \tag{26}$$

2. *War can be avoided when war necessarily occurs in an MPCE, and country S can have a higher utility relative to the MPCE,*

3. *War occurs with probability 1 if preferences satisfy* (17) *for* $\sigma < 1$ *and*

$$-\beta l(m^*(e)) < \psi(1-\beta) - \left(1 - w(m^*(e))^{1-1/\sigma}\right) V(e) \;\; \forall e \leq e_0, \text{ and} \tag{27}$$

4. *War can occur with probability 1 when war is necessarily avoided in the MPCE, and country S can have a lower utility than in the MPCE.*

Proof. See Appendix. ■

The first part of the proposition shows that, under some circumstances, the ability for country S to control resource extraction allows it to avoid wars in situations in which the cost of armament is bounded below the cost of war. For instance if $\psi = -\infty$, so that war is infinitely costly to country S, then country S avoids war in any monopolistic equilibrium and this is true even though wars may be inevitable in the competitive equilibrium. Similarly, if $\bar{m} < \infty$, war does not take place in MPME for large but finite ψ. The second part of the proposition is a simple consequence of the first. When war is highly costly the country S and takes place in the competitive environment but not in the monopolistic environment, then country S's utility will clearly be higher in the monopolistic environment.

Nonetheless, parts 3 and 4 of the proposition show that the opposites of these conclusions might also be true. In particular, if ψ is sufficiently low, offers necessary to secure peace may be very costly for country S, especially since it is implicitly paying for the costs of future armament. In this case, wars can occur along the equilibrium path. More specifically, in contrast to Section 3, country A needs to make costly investments in armament in each period, *even if war does not take place*. This is because, as we noted above, country S cannot commit to making attractive offers unless country A has an effective threat of war, and thus country A is induced to invest in armaments to improve its terms of trade. But this means that war will reduce future armored costs, and thus to secure peace, country S must make offers that compensate country A for the costs of future armament. If these costs are increasing to infinity along the equilibrium path, then the cost to country S of such offers may eventually exceed the cost from war, $-\psi$, which means that war cannot be permanently avoided. More generally, this cost of war may be sufficiently low that country S prefers to allow immediate war in the monopolistic equilibrium even though war does not occur in the competitive equilibrium.

In sum, allowing country S to control the extraction of resource introduces two new economic forces relative to the competitive environment. On the one hand, it implies that country S controls the externalities generated by competitive firms. On the other hand, it also introduces strategic interactions between the two countries, particularly related to country S's inability to commit to making attractive offers to country A without armaments by the latter. This lack of commitment implies that country A will have an incentive to use investments in armaments in order to enhance its terms of trade, even when war will not take place along the equilibrium path. The first force implies that war can be avoided or delayed in the monopolistic equilibrium in situations in which it is inevitable in the competitive equilibrium. The second force implies that war takes place in the monopolistic equilibrium in situations in which it does not occur in the competitive equilibrium since country A must now invest in armament under peace.

5 Extensions

We consider several extensions of the monopolistic environment in order to highlight the robustness of our main result in Propositions 4 and 5. To simplify our discussion, we assume $\psi \to -\infty$ so that wars never occur along the equilibrium path.

5.1 Armament in Defense

In practice, a defending country S can also invest in armament in order to deter an attack. In this section, we consider an extension of our framework which allows country S to also invest in armament.

Formally, at every t, country S can invest in armament $m_{St} \geq 0$ which costs $l(m_{St})$ whereas country A invests in armament $m_{At} \geq 0$ which costs $l(m_{At})$ as before. Country S continues to achieve a payoff ψ in the event of war, though country A now receives a fraction of the remaining endowment $w(m_{At}, m_{St})$ which is increasing and concave in m_{At} and decreasing an convex in m_{St} with $\lim_{m_{At}\to 0} w_{m_{At}}(m_A, m_S) = -\lim_{m_{St}\to 0} w_{m_{St}}(m_A, m_S) = \infty$.

The order of events at t if $f_{t-1} = 0$ is exactly as in Section 4 with the exception that in the first stage, countries A and S simultaneously choose m_{At} and m_{St}, respectively. Using this framework, we can define the MPME as in Section 4 with $U_A(e_t)$ and $U_S(e_t)$ denoting the continuation values to countries A and S, respectively, given endowment e_t.

By the same reasoning as in Section 4, at all t country A chooses the level of armament which maximizes its payoff from war in order to receive the most favorable offer from country S. More specifically, it must be that in equilibrium $m_{At} = \widetilde{m}_A^*(e_t, m_{St})$ for

$$\widetilde{m}_A^*(e_t, m_{St}) = \arg\max_{m_A} V(w(m_A, m_{St})e_t) - l(m_A).$$

Given our assumptions on $u(\cdot)$, $w(\cdot)$, and $l(\cdot)$, $\widetilde{m}_A^*(e_t, m_{St}) > 0$ and is a continuously differentiable function in all of its elements. Since country S makes country A indifferent to going to war, this implies an analogous equation to (21):

$$U_A(e_t) = V(w(\widetilde{m}_A^*(e_t, m_{St}), m_{St})e_t) - l(\widetilde{m}_A^*(e_t, m_{St})).$$

Moreover, analogous arguments to those of Section 4 imply that if $\tilde{U}_S(e_t; m_{At}; m_{St})$ corresponds to country S's welfare from its optimal offer conditional on e_t, m_{At}, and m_{St}, then it must satisfy:

$$\tilde{U}_S(e_t; m_{At}; m_{St}) = \max_{x_t \geq 0, c_t} \{-c_t - l(m_{St}) + \beta U_S(e_{t+1})\} \tag{28}$$

s.t. (3), and

$$u(x_t) + c_t - l(m_{At}) + \beta U_A(e_{t+1}) \geq V(w(m_{At}, m_{St})e_t) - l(m_{At}). \tag{29}$$

Given that $\psi = -\infty$, country S always makes an offer which is accepted and it is the case that $U_S(e_t) = \tilde{U}_S(e_t; m_{At}, m_{St})$. Since the optimal offer lets (29) bind, substitution of (29) into (28) implies that the optimal value of m_{St} which maximizes $U_S(e_t)$ satisfies $m_{St} = \widetilde{m}_S^*(e_t, m_{At})$ for

$$\widetilde{m}_S^*(e_t, m_{At}) = \arg\max_{m_S} -V(w(m_{At}, m_{St})e_t) - l(m_S)$$

for $m_S^*(e_t, m_{At}) > 0$ which is also continuously differentiable.

Note that given this formulation, it is not necessarily always going to be the case in an MPME that $U_A(e_t)$ is differentiable as in Section 4. To facilitate the analysis, we therefore only focus here on MPME for which $U_A(e_t)$ is differentiable. This means that this environment one can define $\{m_A^*(e_t), m_S^*(e_t)\}$ which represents two continuously differentiable functions which correspond to the equilibrium level of armament for each country conditional on the endowment e_t.

Proposition 7 *In any MPME,*

1. *Resource extraction satisfies*

$$\begin{aligned}
\beta u'(x_{t+1}) &> u'(x_t) \text{ if } l'(m_A^*(e_t))\, m_A^{*\prime}(e_{t+1}) + l'(m_S^*(e_t))\, m_S^{*\prime}(e_{t+1}) > 0, \text{ and} \\
\beta u'(x_{t+1}) &< u'(x_t) \text{ if } l'(m_A^*(e_t))\, m_A^{*\prime}(e_{t+1}) + l'(m_S^*(e_t))\, m_S^{*\prime}(e_{t+1}) < 0, \text{ and}
\end{aligned}$$

2. If u satisfies

$$-u'(x) / \left(xu''(x)\right) > (<)\, 1 \text{ for all } x,$$

then $m_A^{*\prime}(e_t) > (<)\, 0$ *and* $m_S^{*\prime}(e_t) > (<)\, 0$.

Proof. See Appendix. ■

Proposition 7 states that the shadow value of the resource rises slower (faster) relative to the Hotelling rule if both $m_A^{*\prime}(e_{t+1})$ and $m_S^{*\prime}(e_{t+1})$ rise (decline) as the resource is depleted. The intuition for this result is analogous to that of Proposition 4, with the exception that in addition to considering the future armament of country A, country S depletes resources taking into account how future values of the endowment will affect its own armament, both directly by changing incentives to arm holding country A's armament fixed and also indirectly by changing country A's armament which co-moves with its own armament.

The second part of the proposition states that if the elasticity of demand exceeds (is below) 1, then the armaments of both country A and country S decline (rise) as the resource is depleted along the equilibrium path. The intuition for this result is the same as that for Proposition 5, with the exception that it takes into account how country A and country S are choosing armaments which optimally react to each other.

5.2 Competing Countries

In practice, international conflict over resources can involve multiple competing resource-poor countries. In this section we consider the implications of allowing for N resource-poor countries labeled by $i = \{1, ..., N\}$ which compete over the resources from country S. The economy is isomorphic to that of Section 4, though the resource constraint is replaced by

$$e_{t+1} = e_t - \sum_{i=1}^{N} x_{it}, \tag{30}$$

where $x_{it} \geq 0$ corresponds to the consumption of the resource by the households (each of mass 1) in country i and $c_{it} \gtreqless 0$ again refers to the consumption good. The flow utility to country i from its consumption of the resource and the consumption good is equal to $u(x_{it}) + c_{it}$ and it discounts the future at the rate β. As such, country S's flow utility from the consumption good equals $\sum_{i=1}^{N} -c_{it}$ and it discounts the future at the rate β.

At any date t, country i can invest in armament $m_{it} \geq 0$ which costs $l(m_{it})$, and it declare a war. The continuation value to country i from war is equal to $V(w_i(m_{it}, \mathbf{m}_{-it})\, e_t) - l(m_{it})$ for $V(\cdot)$ defined as in (4) for a given function $w_i(\cdot) \in [0, 1]$ which is increasing and concave in m_{it} but decreasing and convex in every element of the vector $\mathbf{m}_{-it} = \{m_{jt}\}_{j=1, j\neq i}^{N}$. We assume that $w_i(\cdot)$ satisfies $\lim_{m_{it}\to 0} w_{im_{it}}(m_{it}, \mathbf{m}_{-it}) = \infty$. The interpretation of $V(\cdot)$ and $w_i(\cdot)$ is that in the event of a war, country i seizes a fraction $w_i(\cdot)$ of the remaining endowment of the resource stock which is increasing in country i's armament and decreasing in the armament of country i's rivals. This outcome can emerge because of a world war in which all countries fight each other. Given this interpretation, we let $f_T = 0$ denote that no war has been declared by any country in periods $t = 0, ..., T$, and we let $f_T = 1$ denote that war has been declared by some country in period $t \leq T$.[14]

At every date t, country S's government publicly makes a take-it-or-leave-it offer to each country i, $\left\{x_{it}^o, c_{it}^0\right\}$, consisting of a quantity of resource to be traded in exchange of the consumption good for each i. For simplicity, we assume that rejection of the offer by any country i automatically leads to world war.

The order of events for all periods t for which $f_{t-1} = 0$ is as follows:

1. Each country i government chooses a level of armament m_{it}.

2. Country S's government makes a take-it-or-leave-it offer $\left\{x_{it}^o, c_{it}^0\right\}$ to each i.

3. Each country i government decides whether or not to declare world war.

4. Consumption takes place.

Using this framework, we can define the MPME as in Section 4. We define $U_i(e_t)$ the continuation value to country i conditional on e_t and $f_{t-1} = 0$ and we define $U_S(e_t)$ analogously for country S. Since $\psi = -\infty$, war is always avoided along the equilibrium path.

By the same reasoning as in Section 4, at all t country i chooses the level of armament which maximizes its payoff from war in order to receive the most favorable offer from country S. More specifically, it must be that in equilibrium $m_{it} = \widetilde{m}_i^*(e_t, \mathbf{m}_{-it})$ for

$$\widetilde{m}_i^*(e_t, \mathbf{m}_{-it}) = \arg\max_{m_i} V\left(w_i\left(m_i, \mathbf{m}_{-it}\right) e_t\right) - l\left(m_i\right).$$

Given our assumptions on $u(\cdot)$, $w_i(\cdot)$, and $l(\cdot)$, $\widetilde{m}_i^*(e_t, \mathbf{m}_{-it}) > 0$ and is a continuously differentiable function in all of its elements. This implies an analogous equation to (21):

$$U_i(e_t) = V\left(w_i\left(\widetilde{m}_i^*(e_t, \mathbf{m}_{-it}), \mathbf{m}_{-it}\right) e_t\right) - l\left(\widetilde{m}_i^*(e_t, \mathbf{m}_{-it})\right)$$

for all i where $m_{jt} = \widetilde{m}_j^*(e_t, \mathbf{m}_{-jt})$ for all j. Note that given this formulation, it is not necessarily always going to be the case in an MPME that $U_i(e_t)$ is continuously differentiable as in the case of Section 4. To facilitate the analysis, we therefore only focus here on MPME for which $U_i(e_t)$ is differentiable. This means that in this environment one can define $\{m_i^*(e_t)\}_{i=1}^N$ which represents N continuously differentiable functions which correspond to the equilibrium level of armament for each country i conditional on the endowment e_t.

Proposition 8 *In any MPME,*

[14] Our analysis can also be interpreted as applying to a situation in which only country i attacks country S and it seizes a fraction of the oil which is decreasing in the armament of its rivals.

1. *Resource extraction satisfies*

$$\beta u'(x_{it+1}) \quad > \quad u'(x_{it}) \;\; \text{if} \; \sum_{j=1}^{N} l'\left(m_j^*(e_{t+1})\right) m_j^{*\prime}(e_{t+1}) > 0 \;\forall j \; \text{and}$$

$$\beta u'(x_{it+1}) \quad < \quad u'(x_{it}) \;\; \text{if} \; \sum_{j=1}^{N} l'\left(m_j^*(e_{t+1})\right) m_j^{*\prime}(e_{t+1}) < 0 \;\forall j, \; \text{and}$$

2. *If u satisfies*

$$-u'(x) / \left(xu''(x)\right) > (<)\, 1 \; \text{for all} \; x,$$

then $m_j^{*\prime}(e_t) > (<)\, 0$ *for all* j.

Proof. See Appendix. ■

Proposition 8 states that the shadow value of resources in country i grows faster or slower than the inverse rate of time preference depending on whether the level of armament *for all countries* is increasing or decreasing in the resource endowment. The intuition for Proposition 8 is similar to that of Proposition 4 with the exception that Proposition 8 takes into account how future values of the endowment e_{t+1} affect the armament of all countries jointly. In other words, it takes into account the strategic behavior of countries vis a vis each other.

More specifically, countries will have an incentive to arm more (less) if their rivals are also arming more (less), and this implies that armaments around the world will co-move as the endowment depletes. The second part of Proposition 8 which is analogous to Proposition 5 states that whether armament increases or decreases as the endowment is depleted depends on the elasticity of demand. The intuition for this is the same in the case of one resource-poor country, the magnitude of this change is now also affected by the change in the armament behavior of neighbors.

To illustrate the complementarity in armament decisions across countries, it is useful to consider a simple example in which we can explore the consequences of changing the number of competing countries N. Suppose that preferences satisfy (17) so that the elasticity of demand is constant. Moreover, suppose that the outstanding stock of the resource at t is Ne_t. This normalization implies that the per capita level of the resource is constant and it allows us to consider how armament decisions change as the level of competition changes.[15] Moreover, let $w_i(\cdot)$ and $l(\cdot)$ take the following functional forms:

$$w_i(m_{it}, \mathbf{m}_{-it}) = \frac{m_{it}}{\sum_{j=1}^{N} m_{jt}} \text{ and } l(m_{it}) = m_{it}. \tag{31}$$

[15] Our results do not depend on this normalization, but we impose it here in order to make the intuition simpler to understand since it implies that the sensitivity of $m_i^*(e_t)$ with respect to e_t rises in N.

In this environment, it can be shown that $m_i^*(e_t)$ satisfies:

$$m_i^*(e_t) = \left(\frac{N-1}{N}\right)(1-\beta^\sigma)^{-1/\sigma} e_t^{1-1/\sigma}, \tag{32}$$

so that it is increasing in the level of competition N. Intuitively, if more countries compete, returns to arming are higher and these returns become more sensitive to changes in the endowment. Therefore, country S takes this into account in deciding the time path of extraction.

Proposition 9 *Under* (17) *and* (31)*,*

1. $u'(x_{it+1}) = (1/\rho)\, u'(x_{it})$ *for all* t*,*
2. $1/\rho > (<)\, 1/\beta$ *if* $\sigma > (<)\, 1$*, and*
3. $|\rho - \beta|$ *is rising in* N *if* $\sigma \neq 1$*.*

Proof. See Appendix. ■

This proposition states that under (17) and (31) the growth rate of the shadow value of the resource is constant and depends on the elasticity of substitution σ. Interestingly, the last part of this lemma states that the distortion in this growth rate from the Hotelling rule is *increasing* in the level of international competition. The intuition for this is that as N rises, the marginal benefit of armament rises so that global armament becomes more sensitive to changes in the resource endowment. For instance, if $\sigma < 1$ so that armament is rising along the equilibrium path, an increase in the level of international competition increases the incentive for country S to slow down the rate of resource depletion so as to mitigate the rise in armament. This occurs precisely because complementarities in global armament rises as international competition increases.

5.3 Alternative Preferences

A natural question is the extent to which our conclusions depend on our assumption of quasi-linear preferences for country A. In this section, we show that our general conclusion embedded in Proposition 4 holds. More specifically, consider an environment in which the flow utility to country A is equal to

$$u(x_t, c_t, -m_t),$$

where $u(\cdot)$ is increasing and globally concave in x_t, c_t, and $-m_t$. Let $\lim_{x\to 0} u_x(\cdot) = \infty$ and $\lim_{x\to\infty} u_x(\cdot) = 0$. For simplicity, we assume that $u(\cdot)$ is defined for all values of $c_t \gtreqless 0$.

Note that in this environment, the Hotelling rule can be written as:

$$u_x(x_{t+1}, c_{t+1}, -m_{t+1}) / u_c(x_{t+1}, c_{t+1}, -m_{t+1}) = (1/\beta)\, u_x(x_t, c_t, -m_t) / u_c(x_t, c_t, -m_t),$$

so that the marginal rate of substitution between the resource and the consumption good rises at the rate of interest.

Consider the order of events and define the MPME as in Section 4. In this environment, we can define:

$$\widetilde{V}(e_t) = \max_{\{x_{t+k}, e_{t+k+1}\}_{k=0}^{\infty}, m_t} u(c_t, 0, -m_t) + \sum_{k=1}^{\infty} \beta^k u(x_{t+k}, 0, 0) \text{ s.t. } (5) - (\textbf{??}).$$

$\widetilde{V}(e_t)$ corresponds to the highest continuation value that country A can achieve in the event of war and it is analogous to $V(w(m^*(e_t))e_t) - l(m^*(e_t))$ in the quasi-linear case. Let $m^*(e_t)$ correspond to the value of m_t associated with $\widetilde{V}(e_t)$.

Proposition 10 *In an MPME,*

$$u_x(x_{t+1}, c_{t+1}, -m_{t+1}) / u_c(x_{t+1}, c_{t+1}, -m_{t+1}) > (<) (1/\beta) u_x(x_t, c_t, -m_t) / u_c(x_t, c_t, -m_t)$$

if

$$m^{*\prime}(e_{t+1}) + \frac{\widetilde{V}'(e_{t+1})}{u_m(x_{t+1}, c_{t+1}, -m_{t+1})}\left(1 - \frac{u_c(x_{t+1}, c_{t+1}, -m_{t+1})}{u_c(x_t, c_t, -m_t)}\right) > (<) 0.$$

Proof. See Appendix. ■

Analogously to Proposition 4, Proposition 10 states that the shadow price of the resource will rise faster (slower) if armament is increasing (decreasing) in the size of the total resource endowment. Nevertheless, in relating this rate of growth to the inverse discount factor, Proposition 10 differs from Proposition 4 because the rate of growth of the shadow price does not only depend on $m^{*\prime}(e_{t+1})$ but also on an additional term which equals zero if preferences are quasi-linear. This term emerges precisely because even in the absence of endogenous armament, distortions in the growth rate of the shadow price can emerge if the marginal utility of the consumption good is time varying. Intuitively, it is cheaper for country S to extract payments from country A while still inducing it away from war if country A's marginal utility from the consumption good is lower. Therefore, if the marginal utility of the consumption good is higher (lower) today relative to tomorrow, then country S will deplete more (less) of the endowment today. Proposition 10 therefore shows that in addition to this force, the sign of $m^{*\prime}(e_{t+1})$ continues to play the same role as in the quasi-linear case. Note that one can conjecture that in a richer environment with additional smoothing instruments such as bonds, this marginal utility of consumption will not vary significantly along the equilibrium path so that the dominating effect would emerge from the direction of $m^{*\prime}(e_{t+1})$.

6 Conclusion

This paper analyzed a dynamic environment in which a resource-rich country trades an exhaustible resource with a resource-poor country. In every period, the resource-poor country can arm and attack the resource-rich country. When the resource is extracted by price-taking firms, in addition to the standard pecuniary externality across firms, there is a novel externality as each firm fails to internalize the impact of their extraction on military action by the resource-poor country. In the empirically relevant case where the demand for the resource is inelastic and the resource-poor country can capture most of the remaining endowment in a war, this makes war inevitable. Because the anticipation of future war encourages more rapid extraction, equilibrium war happens in the initial period.

Externalities across price-taking firms can be internalized by the government of the resource-rich country regulating the price and the level of production of the resource. This "monopolistic" environment can prevent or delay wars when they occur immediately under competitive markets. The resource-rich country does so by making offers that leave the resource-poor country indifferent between war and peace at each date. Interestingly, this involves a deviation from the Hotelling rule, because depending on whether incentives for war are increasing or decreasing in the remaining endowment of the resource, the resource-rich country prefers to adopt a slower or more rapid rate of extraction of the resource than that implied by the Hotelling rule. In particular, in the empirically relevant case where the demand elasticity for the resource is less than one, extraction is slower and resource prices increase more slowly than under the Hotelling rule because this enables the resource-rich country to slow down the increase in armaments, for which it is paying indirectly. Conversely, when demand is elastic, the resource-rich country can reduce armaments cost by adopting a more rapid path of resource extraction than the one implied by Hotelling rule.

However, a naive conjecture that regulation of prices and quantities by the resource-rich country will necessarily prevent war and make its citizens necessarily better off is also incorrect. The monopolistic environment, which allows for such regulation and in fact gives the resource-rich country the ability to make take-it-or-leave-it offers, leads to a different type of distortion: because the resource-rich country cannot commit to making attractive offers to the resource-poor country without the latter arming, the equilibrium path involves armaments at each date. The resource-rich country must then, implicitly, pay the future costs of armaments in order to prevent war. This might, paradoxically, make war more likely.

Finally, we also show that the main insights generalize when there are several countries competing for resources and when the resource-rich country can also invest in armaments for defense.

We view our paper as a first step in the analysis of interactions between dynamic trade and inter-country military actions. These ideas appear particularly important in the context of natural resources since trade is necessarily dynamic and international trade in natural resources has historically been heavily affected by military conflict or the threat thereof. Despite the simplicity of the economic environment studied here, both under competitive markets and when the resource-rich country can regulate prices and quantities, there are rich interactions between economic equilibria and international conflict. Both the path of prices is affected by the future probabilities of war, and war is determined by the paths of prices and quantities. We think that further study of dynamic interactions between trade, international conflict and political economy is a fruitful area for future research.

7 Appendix [Incomplete]

7.1 Proofs from Section 3

Definition of Strategies at $e_t = 0$ for $u(0) = -\infty$

If the endowment equals 0, then the payoff from war and from peace both equal $-\infty$. We determine whether or not war occurs in this case by using limit arguments. Specifically, let

$$U^C(e) = \sum_{t=0}^{\infty} \beta^t \left(u(\widetilde{x}_t(e)) - u'(\widetilde{x}_t(e)) \widetilde{x}_t(e) \right)$$
$$\text{for } \{\widetilde{x}_t(e), \widetilde{e}_t(e)\}_{t=0}^{\infty} \text{ which satisfies}$$
$$u'(\widetilde{x}_{t+1}(e)) = (1/\beta) u'(\widetilde{x}_t(e)),$$
$$\widetilde{e}_{t+1}(e) = \widetilde{e}_t(e) - \widetilde{x}_t(e), \text{ and } \widetilde{e}_0(e) = e.$$

$U^C(e)$ corresponds to equilibrium welfare of country A in a permanently peaceful competitive equilibrium starting from endowment e at date 0, where $\widetilde{x}_t(e)$ and $\widetilde{e}_t(e)$ correspond to the resource consumption and resource endowment, respectively, at date t in such an equilibrium.

Define

$$F(e) = U^C(e) - (V(w(m^*(e))e) - l(m^*(e)) - v). \tag{33}$$

$F(e)$ corresponds to the difference in country A's welfare between a permanently peaceful competitive equilibrium and war with optimal armament $m^*(e)$ starting from endowment e. We define the behavior of country A starting from 0 endowment in terms of the limiting behavior of $F(e)$.

Definition 3 *If $f_{t-1} = 0$ and $e_t = 0$, then country A chooses $f_t = 0$ if $\lim_{e \to 0} F(e) \geq 0$, otherwise country A chooses $f_t = 1$.*

Proof of Lemma 1

We prove existence of an MPCE for three separate cases: (i) $u(0) = -\infty$ and $\lim_{e\to 0} F(e) < 0$, the case in which there is infinite disutility from zero oil consumption and country A goes to war if there is zero oil endowment; (ii) $\lim_{e\to 0} F(e) \geq 0$, and $F(e) \geq 0\ \forall e \leq e_0$, the case in which country A does not go to war if there is zero oil endowment and country A weakly prefers permanent peace to immediate war for *all* feasible endowment levels; and (iii) $\lim_{e\to 0} F(e) \geq 0$ and $F(e) < 0$ for some $e \leq e_0$, the case in which country A does not go to war if there is zero oil endowment and country A prefers immediate war to permanent peace for *some* feasible endowment levels. For the first two cases, we prove the existence of equilibrium in pure strategies, and in the third case, we use the possibility of mixed strategies to prove existence.

Part 1. If $u(0) = -\infty$ and $\lim_{e\to 0} F(e) < 0$, then by definition, $f_t = 1$ if $e_t = 0$, so that country A goes to war if there is zero endowment. We can construct an equilibrium with immediate war at date 0 as follows. If firms expect country A to declare war at date 1, then they extract the entire endowment of oil at date 0. This leads to zero oil endowment at date 1 in the event of peace at date 0, which implies that country A would in fact go to war at date 1, verifying the expectation of the firms. Taking this into account, at date 0, country A chooses between going to immediate war versus having peace at date 0 followed by war at date 1. Under the latter option, country A receives infinite disutility from consuming zero oil from date 1 onward, and this implies that it strictly prefers going to war at date 0 versus postponing war to date 1. Formally, let $\left(e_0^*, p_0^*, x_0^{S*}, x_0^{A*}\right) = \left(e_0, u'\left(e_0\right), e_0, e_0\right)$ and $\left(e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right) = (0, \infty, 0, 0)$ $\forall t > 0$. Define country A's strategy such that conditional on any $e_t > 0$, it chooses $m_t = m^*\left(e_t\right)$ and $f_t = 1$. To check that this constitutes an MPCE, note that $\left\{e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right\}_{t=0}^{\infty}$ satisfies (3), (10), (12), and (13). To check that country A's strategy is optimal note that the only point in the sequence γ with $e_t^* > 0$ is e_0. This means that we need only check optimality of its decision at date 0 of letting $m_0 = m^*\left(e_0\right)$ and $f_0 = 1$. By definition, conditional on choosing $f_0 = 1$, $m_0 = m^*\left(e_0\right)$ is optimal. If instead country A chose $f_0 = 0$, then it would receive $x_t = 0$ $\forall t \geq 1$, achieving a continuation payoff at date 0 of $-\infty$ which is strictly dominated by $V\left(w\left(m^*\left(e_0\right)\right)e_0\right) - l\left(m^*\left(e_0\right)\right) - \upsilon > -\infty$. Therefore, country A's strategy is optimal, verifying the existence of this MPCE.

Part 2. If $\lim_{e \to 0} F\left(e\right) \geq 0$, and $F\left(e\right) \geq 0$ $\forall e \leq e_0$, then country A does not go to war if there is zero oil endowment and country A weakly prefers permanent peace to immediate war for all feasible endowment levels. It is straightforward to see that this implies that one can construct an equilibrium with permanent peace since permanent peace weakly dominates immediate war for all equilibrium endowment levels. Formally, $\left\{e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right\}_{t=0}^{\infty} = \left\{\widetilde{e}_t\left(e_0\right), u'\left(\widetilde{x}_t\left(e_0\right)\right), \widetilde{x}_t\left(e_0\right), \widetilde{x}_t\left(e_0\right)\right\}_{t=0}^{\infty}$. Define country A's strategy such that conditional on any $e_t > 0$, it chooses $m_t = 0$ and $f_t = 0$. To check that this constitutes and MPCE, note that $\left\{e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right\}_{t=0}^{\infty}$ satisfies (3), (10), (12), and (13). To check that country A's strategy is optimal, note that since $F\left(e\right) \geq 0$ $\forall e$, then for any e_t^* in γ country A weakly prefers $f_t = 0$ under optimal armament $m_t = 0$ to $f_t = 1$ under optimal armament $m_t = m^*\left(e_t\right)$ so that its strategy is optimal. This verifies the existence of an MPCE in this case.

Part 3. If $\lim_{e \to 0} F\left(e\right) \geq 0$, and $F\left(e\right) < 0$ for some $e \leq e_0$, then country A does not go to war if there is zero oil endowment and country A prefers immediate war to permanent peace for some feasible endowment levels. This case is the most complicated for the following reasons. Because $F\left(e\right) < 0$ for some $e \leq e_0$, a permanently peaceful equilibrium as that constructed in part 2 may not be incentive compatible for country A since it may prefer immediate war for some equilibrium endowment level. Moreover, because $\lim_{e \to 0} F\left(e\right) \geq 0$, an equilibrium with immediate war as that constructed in part 1 does not exist. Specifically, if firms expecting war at date 1 extract the entire endowment at date 0, then at date 1 country A optimally chooses not to go to war since there is zero endowment, which violates the firms' expectations of war at date 1. In proving the existence of equilibrium, we proceed in three steps. First, we define a point $\widehat{e}$, and we argue that we need only consider proving existence for initial endowments e_0

which exceed $\widehat{e}$. Second, we define a point $\widehat{\widehat{e}} > \widehat{e}$, and we prove the existence of equilibrium for $e_0 \in \left(\widehat{e}, \widehat{\widehat{e}}\right]$. Third, we prove the existence of equilibrium for $e_0 > \widehat{\widehat{e}}$.

First, we show that there exists a threshold $\widehat{e} > 0$ with the following feature: if $e_0 \leq \widehat{e}$, then we are in the case of part 2 and there is an equilibrium with permanent peace, and if $e_0 = \widehat{e}$, country A is indifferent between permanent peace and immediate war. This means that the relevant range of endowments to consider is for $e_0 > \widehat{e}$. Formally, define $\widehat{e} > 0$ such that $F(\widehat{e}) = 0$ and $F(e) \geq 0 \; \forall e \in [0, \widehat{e}]$. To see that such a point $\widehat{e} > 0$ exists, note that if $u(0) > -\infty$, then $F(0) = \upsilon > 0$ and $F(e) < 0$ for some e which by continuity implies the existence of $\widehat{e} > 0$. Alternatively, if $u(0) = -\infty$, then if it were the case that $\lim_{e \to 0} F(e) = 0$ for some υ, then this would imply that $\lim_{e \to 0} F(e) < 0$ under some $\upsilon' = \upsilon - \epsilon$ for $\epsilon > 0$ chosen arbitrarily small, which is the relevant case here since we characterize equilibria as $\upsilon \to 0$. Therefore, in this case $\lim_{e \to 0} F(e) > 0$ and $F(e) < 0$ for some e which by continuity implies the existence of $\widehat{e} > 0$. Because $F(e) \geq 0 \; \forall e \in [0, \widehat{e}]$, if $e_0 \leq \widehat{e}$ then the results of part 2 apply. Therefore, we only consider $e_0 > \widehat{e}$.

Second, we show that there exists a threshold $\widehat{\widehat{e}} > \widehat{e}$, and we characterize equilibrium for $e_0 \leq \widehat{\widehat{e}}$. We show that if $e_0 \leq \widehat{\widehat{e}}$, then the absence of war at date 0 leads to permanent peace from date 1 onward since firms extract a sufficient level of oil at date 0 that the date 1 endowment is below $\widehat{e}$, the threshold necessary to embark on a permanently peaceful equilibrium. Taking this into account, country A decides at date 0 whether it prefers permanent peace to immediate war, and the ensuing equilibrium depends on whether $F(e_0)$ exceeds or is below 0. Formally, define $\widehat{\widehat{e}} > \widehat{e}$ for which $\widetilde{e}_1\left(\widehat{\widehat{e}}\right) = \widehat{e}$. $\widehat{\widehat{e}}$ represents an initial endowment at date 0 from which firms extract the level of resources which leads the economy to an endowment of $\widehat{e}$ at date 1. $\widehat{\widehat{e}}$ is uniquely defined since $\widetilde{e}_1(\cdot)$ is an increasing function with $\widetilde{e}_1(\widehat{e}) > \widehat{e}$ for all $\widehat{e} > 0$. If $e_0 \leq \widehat{\widehat{e}}$, then let $\left\{e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right\}_{t=0}^{\infty} = \left\{\widetilde{e}_t(e_0), u'(\widetilde{x}_t(e_0)), \widetilde{x}_t(e_0), \widetilde{x}_t(e_0)\right\}_{t=0}^{\infty}$. Define country A's strategy such that conditional on any $e_t \leq \widehat{e}$, it chooses $m_t = 0$ and $f_t = 0$. Since $e_t^* \leq \widehat{e} \; \forall t \geq 1$, we are left to specify country A's strategy at e_0. If $F(e_0) \geq 0$, then it chooses $m_0 = 0$ and $f_0 = 0$ at e_0, otherwise it chooses $m_0 = m^*(e_0)$ and $f_0 = 1$. To check that this constitutes an MPCE, note that $\left\{e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right\}_{t=0}^{\infty}$ satisfies (3), (10), (12), and (13). To check that country A's strategy is optimal, note that since $F(e) \geq 0 \; \forall e \leq \widehat{e}$, then country A weakly prefers $f_t = 0$ under optimal armament $m_t = 0$ to $f_t = 1$ under optimal armament $m_t = m^*(e_t)$ so that its strategy is optimal for $e_t \leq \widehat{e}$. Analogous reasoning implies that if $F(e_0) \geq 0$, then $m_0 = 0$ and $f_0 = 0$ is a weakly dominant strategy, otherwise $m = m^*(e_0)$ and $f_0 = 1$ is optimal. This verifies the existence of an MPCE in this case for $e_0 \leq \widehat{\widehat{e}}$.

Finally, we characterize the equilibrium for $e_0 > \widehat{\widehat{e}}$. If $e_0 > \widehat{\widehat{e}}$, then under a permanently peaceful equilibrium, firms would extract a level of resources at date 0 which would lead to $e_1 > \widehat{e}$, a point from which the equilibrium cannot be easily characterized. For this reason, we construct an equilibrium which leads firms to extract a level of resources at date 0 which leads to $e_1 = \widehat{e}$ at date 1 in the absence of war at date 0. This is achieved by having the firms expect country A to mix between war and peace at date 0, and this is incentive compatible for country A since $F(e_1) = 0$ so that country A is indifferent between immediate war and permanent peace. At date 1, country A mixes between war and peace, and if war is avoided at date 1, there is permanent peace from date 2 onward. Taking this into account at date 0, country A decides whether it prefers immediate war or the continuation equilibrium just described. Formally, let $\left(e_0^*, p_0^*, x_0^{S*}, x_0^{A*}\right) = \left(e_0, u'(e_0 - \widehat{e}), e_0 - \widehat{e}, e_0 - \widehat{e}\right)$ and $\left(e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right) = \left(\widetilde{e}_{t-1}(\widehat{e}), u'(\widetilde{x}_{t-1}(\widehat{e})), \widetilde{x}_{t-1}(\widehat{e}), \widetilde{x}_{t-1}(\widehat{e})\right) \forall t \geq 1$. Define country A's strategy such that conditional on any $e_t < \widehat{e}$, it chooses $m_t = 0$ and $f_t = 0$. Since $e_t^* < \widehat{e}\ \forall t \geq 2$, we are left to specify country A's strategy at $e_1^* = \widehat{e}$ and e_0. At $\widehat{e}$, country A chooses $m_1 = 0$ and $f_1 = 0$ with probability $u'(e_0 - \widehat{e}) / (\beta u'(\widetilde{x}_0(\widehat{e}))) \in (0,1)$ and $m_1 = m^*(\widehat{e})$ and $f_1 = 1$ with probability $1 - u'(e_0 - \widehat{e}) / (\beta u'(\widetilde{x}_0(\widehat{e})))$. At e_0, if

$$u(e_0 - \widehat{e}) - u'(e_0 - \widehat{e})(e_0 - \widehat{e}) + \beta U^C(\widehat{e}) \geq V(w(m^*(e_0))e_0) - l(m^*(e_0)) - \upsilon, \tag{34}$$

then country A chooses $m_0 = 0$ and $f_0 = 0$, otherwise it chooses $m_0 = m^*(e_0)$ and $f_0 = 1$. To check that this constitutes an MPCE, note that $\left\{e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right\}_{t=0}^{\infty}$ satisfies (3), (10), (12), and (13). Analogous reasoning to the case with $e_0 \leq \widehat{\widehat{e}}$ implies that country A's strategy is optimal for $e_t^* < \widehat{e}$. Since $F(\widehat{e}) = 0$, starting from $e_1^* = \widehat{e}$, country A is indifferent between $f_1 = 0$ under optimal armament $m_1 = 0$ and $f_1 = 1$ under optimal armament $m_1 = m^*(\widehat{e})$ so that its strategy is optimal. Finally, starting from e_0, country A receives a payoff equal to the left hand side of (34) in choosing $f_0 = 0$ under optimal armament $m_0 = 0$, and it receives a payoff equal to the right hand side of (34) under $f_0 = 1$ and optimal armament $m_0 = m^*(e_0)$. Therefore, if (34) holds then $m_0 = 0$ and $f_0 = 0$ is optimal, otherwise the choice of $m_0 = m^*(e_0)$ and $f_0 = 1$ is optimal. This verifies the existence of an MPCE in this case for $e_0 > \widehat{\widehat{e}}$.■

Proof of Proposition 2

The arguments in the proof of Proposition 1, imply that $\Pr\{f_t = 1 | f_{t-1} = 0\} < 1$, meaning that it is not possible for country A to go to war at some date $t \geq 1$ with probability 1. We therefore are left to prove that it is not possible for country A to go to war at some date $t \geq 1$ with any probability between 0 and 1. Formally, let $\mu_t = \Pr\{f_t = 0 | f_{t-1} = 0\} > 0\ \forall t$ and consider a sequence $\{\mu_t\}_{t=0}^{\infty}$. Given that Proposition 1 implies that $\mu_t > 0$ for $t \geq 1$, we argue by contradiction that $\mu_t \notin (0,1)$ for $t \geq 1$. To establish this result, we first establish the below preliminary lemma which provides a necessary condition which must hold given a sequence $\{\mu_t\}_{t=0}^{\infty}$ with $\mu_t > 0\ \forall t \geq 1$. This lemma provides a necessary condition which must hold if $\mu_t \in (0,1)$ for $t \geq 1$, and we use this lemma to later prove by contradiction that this necessary condition cannot hold.

Lemma 3 *Given $\{\mu_t\}_{t=0}^{\infty}$ with $\mu_t > 0\ \forall t \geq 1$, it is necessary that $\forall t \geq 1$,*

$$K_t e_t^{1-1/\sigma} \geq w(m^*(e_t))^{1-1/\sigma}(1 - \beta^{\sigma})^{-1/\sigma} \frac{1}{1 - 1/\sigma} e_t^{1-1/\sigma} - l(m^*(e_t)) - \upsilon \tag{35}$$

for

$$K_t = \frac{1}{\sigma}\frac{1}{1-1/\sigma}\frac{\left(1+\sum_{k=1}^{\infty}\beta^k\left(\prod_{l=1}^{k}\left(\beta\mu_{t+l}\right)^{\sigma}\right)^{1-1/\sigma}\right)}{\left(1+\sum_{k=1}^{\infty}\prod_{l=1}^{k}\left(\beta\mu_{t+l}\right)^{\sigma}\right)^{1-1/\sigma}}, \tag{36}$$

where (i) K_t is bounded from above and below and (ii) (35) *binds if $\mu_t < 1$.*

Proof. To establish the necessity of (35), we first establish the necessity of the below condition and then argue that it is equivalent to (35):

$$\sum_{k=0}^{\infty}\beta^k\left(u\left(x_{t+k}\right)-u'\left(x_{t+k}\right)x_{t+k}\right) \geq V\left(w\left(m^*\left(e_t\right)\right)e_t\right)-l\left(m^*\left(e_t\right)\right)-\upsilon. \tag{37}$$

We also argue that (37) must bind if $\mu_t < 1$ so that the probability of war is positive, which establishes that (35) binds if $\mu_t < 1$. The left hand side of (37) represents country A's continuation value from permanent peace and the right hand side of (37) represents the continuation value to country A from immediate war. To see why, note that if war does not take place, country A does not arm, and if war takes place, country A chooses optimal armament $m^*(e_t)$. Specifically, if $f_t = 0$, then $m_t = 0$ and if $f_t = 1$, then $m_t = m^*(e_t)$. Suppose it were the case that $f_t = 0$ and $m_t > 0$. Then country A could make itself strictly better off by reducing m_t while still choosing $f_t = 0$. Alternatively, if $f_t = 1$ but $m_t \neq m^*(e_t)$, then country A could make itself strictly better off by choosing $m_t = m^*(e_t)$ and $f_t = 1$. Since country A always weakly prefers peace to immediate war with optimal armament, this implies (37). This is because the left hand side of (37) represents country A's equilibrium continuation value, which must be equal to that from permanent peace with zero armament, taking into account (12) and (13) which can be substituted into (14) for every e_t. This establishes the necessity of (37). If $\mu_t < 1$, then country A chooses war and peace each with positive probability, implying that it must be indifferent between the two options, otherwise it could make itself strictly better off by choosing $\mu_t = 1$. This implies that (37) binds in this case.

We next establish that (37) is equivalent to (35). The right hand side of (37) equals the right hand side of (35) minus $1/\left[\left(1-\beta\right)\left(1-1/\sigma\right)\right]$, and this follows by substitution of (17) into (4). We can now show that the left hand side of (37) equals the left hand side of (35) minus $1/\left[\left(1-\beta\right)\left(1-1/\sigma\right)\right]$. To do this, we determine how the sequence $\{x_t\}_{t=0}^{\infty}$ relates to $\{\mu_t\}_{t=0}^{\infty}$. Optimal extraction for firms requires that

$$\mu_{t+1}p_{t+1} = \frac{1}{\beta}p_t. \tag{38}$$

If instead $\mu_{t+1}p_{t+1} > \frac{1}{\beta}p_t$, then from condition (12) $x_t^A > 0$ since $p_t < \infty$. From (10) $x_t^S = 0$, but this implies that $x_t^S \neq x_t^A$ which violates (13). If instead $\mu_{t+1}p_{t+1} < \frac{1}{\beta}p_t$, then analogous arguments imply that $x_{t+1}^A > 0$ and $x_{t+1}^S = 0$ which violates (13). (38) together with (12) implies that

$$x_{t+1} = \left(\beta\mu_{t+1}\right)^{\sigma} x_t. \tag{39}$$

Given (3), this implies that

$$x_t\left(1 + \sum_{k=1}^{\infty}\prod_{l=1}^{k}\left(\beta\mu_{t+l}\right)^{\sigma}\right) = e_t. \tag{40}$$

Equation (40) together with the fact that $\mu_t > 0\ \forall t > 0$ implies that

$$e_t > 0 \text{ and } \frac{x_t}{e_t} = \frac{1}{1 + \sum_{k=1}^{\infty}\prod_{l=1}^{k}\left(\beta\mu_{t+l}\right)^{\sigma}} \geq 1 - \beta^{\sigma} > 0\ \forall t. \tag{41}$$

Consequently, by substitution, one can write the left hand side of (37) as $K_t e_t^{1-1/\sigma} - 1/\left[(1-\beta)(1-1/\sigma)\right]$ for (36).We are left to show that K_t is bounded. To see that it is bounded from above, note that

$$K_t \leq K^* = \frac{1}{\sigma}\frac{1}{1-1/\sigma}\left(1-\beta^{\sigma}\right)^{-1/\sigma}. \tag{42}$$

This is because substitution of (12) into $U_A\left(e_t\right)$ implies that

$$\begin{aligned} U_A\left(e_t\right) &= \sum_{k=0}^{\infty}\frac{1}{\sigma}\left(\beta^k\frac{x_{t+k}^{1-1/\sigma}}{1-1/\sigma}\right) - 1/\left[(1-\beta)(1-1/\sigma)\right] \\ &\leq \frac{1}{\sigma}\frac{1}{1-1/\sigma}\left(1-\beta^{\sigma}\right)^{-1/\sigma} e_t^{1-1/\sigma} - 1/\left[(1-\beta)(1-1/\sigma)\right], \end{aligned} \tag{43}$$

where the last inequality follows from the maximization of (43) given the resource constraint (3). To see that K_t is bounded from below, note that if $\sigma > 1$, (36) implies that

$$K_t \geq \frac{1}{\sigma}\frac{1}{1-1/\sigma}\left(1-\beta^{\sigma}\right)^{1-1/\sigma}.$$

If instead $\sigma < 1$, then the fact that country A weakly prefers peace to war under any armament level $m > 0$ implies that

$$K_t e_t^{1-1/\sigma} \geq w\left(m\right)^{1-1/\sigma}\left(1-\beta^{\sigma}\right)^{-1/\sigma}\frac{1}{1-1/\sigma}e_t^{1-1/\sigma} - l\left(m\right) - v,$$

which simplifies to

$$K_t \geq w\left(m\right)^{1-1/\sigma}\left(1-\beta^{\sigma}\right)^{-1/\sigma}\frac{1}{1-1/\sigma}-\frac{l\left(m\right)-\upsilon}{e_0^{1-1/\sigma}}$$

which means that K_t is bounded from below. This completes the proof that if $\Pr\{f_t = 1|f_{t-1} = 0\} > 0$, then (35) must hold for sequence $\{K_t\}_{t=0}^{\infty}$ where each element is from a bounded set. ■

We now show that (35) cannot bind for any $t \geq 1$, which proves that it is not possible for $\Pr\{f_t = 1|f_{t-1} = 0\} > 0$ for $t \geq 1$. There are two cases to consider. In the first case, $\mu_T \in (0, 1)$ and $\mu_t = 1\ \forall t \geq T + 1$ so that war stops occurring after some date T. In the second case, there does not exist such a date T and there is always a positive probability of war in the future.

In the first case, since country A is indifferent to war at T and weakly prefers peace at $T - 1$ and $T + 1$ to war using the same armament as at T, it follows that:

$$K_{T+1}e_{T+1}^{1-1/\sigma} \geq w\left(m^*\left(e_T\right)\right)^{1-1/\sigma}\left(1-\beta^{\sigma}\right)^{-1/\sigma}\frac{1}{1-1/\sigma}e_{T+1}^{1-1/\sigma} - l\left(m^*\left(e_T\right)\right)-\upsilon \quad (44)$$

$$K_{T}e_{T}^{1-1/\sigma} = w\left(m^*\left(e_T\right)\right)^{1-1/\sigma}\left(1-\beta^{\sigma}\right)^{-1/\sigma}\frac{1}{1-1/\sigma}e_{T}^{1-1/\sigma} - l\left(m^*\left(e_T\right)\right)-\upsilon \quad (45)$$

$$K_{T-1}e_{T-1}^{1-1/\sigma} \geq w\left(m^*\left(e_T\right)\right)^{1-1/\sigma}\left(1-\beta^{\sigma}\right)^{-1/\sigma}\frac{1}{1-1/\sigma}e_{T-1}^{1-1/\sigma} - l\left(m^*\left(e_T\right)\right)-\upsilon \quad (46)$$

Since $\mu_t = 1\ \forall t \geq T + 1$, from (36), it must be the case that $K_{T+1} = K_T = K^*$ for K^* defined in (42), and since $\mu_T \in (0, 1)$, it must be that $K^* > K_{T-1}$ since war is chosen with positive probability at T. (44) – (46) taking into account that $m^*\left(e_T\right) > 0$ imply that

$$e_T^{1-1/\sigma} - e_{T+1}^{1-1/\sigma} \geq 0 \text{ and } e_T^{1-1/\sigma} - e_{T-1}^{1-1/\sigma} \geq 0.$$

If $\sigma < 1$, then by (3) this implies that $e_{T+1} = e_T$ so that $x_T = 0$ which violates (41). If instead $\sigma > 1$, then this implies that $e_T \geq e_{T-1}$ which implies $x_{T-1} = 0$, which violates (41). This establishes that it is not possible for $\Pr\{f_t = 1|f_{t-1} = 0\} > 0$ for $t \geq 1$ in an equilibrium in which war stops occurring after some date T.

Now consider the second case in which war never stops occurring and consider the implied sequence $S = \{\mu_t, K_t\}_{t=0}^{\infty}$ under the hypothetical MPCE and select the infinite subsequence $s^1 \in S$ for which $\mu_t \neq 1$ so that war occurs with positive probability for all elements of s^1. Since the each element of the sequence of K_t's is in a closed and bounded set, we can select a convergent subsequence s^2 within s^1 for which the K_t's converge. Consider three consecutive elements of s^2, denoted by $n - 1$, n, and $n + 1$. Weak preference for peace at $n - 1$ and $n + 1$

together with indifference to peace at n using armament $m^*(e_n)$ implies:

$$K_{n+1}e_{n+1}^{1-1/\sigma} \geq w(m^*(e_n))^{1-1/\sigma}(1-\beta^\sigma)^{-1/\sigma}\frac{1}{1-1/\sigma}e_{n+1}^{1-1/\sigma} - l(m^*(e_n)) - \upsilon \quad (47)$$

$$K_n e_n^{1-1/\sigma} = w(m^*(e_n))^{1-1/\sigma}(1-\beta^\sigma)^{-1/\sigma}\frac{1}{1-1/\sigma}e_n^{1-1/\sigma} - l(m^*(e_n)) - \upsilon \quad (48)$$

$$K_{n-1}e_{n-1}^{1-1/\sigma} \geq w(m^*(e_n))^{1-1/\sigma}(1-\beta^\sigma)^{-1/\sigma}\frac{1}{1-1/\sigma}e_{n-1}^{1-1/\sigma} - l(m^*(e_n)) - \upsilon \quad (49)$$

(47) and (48) imply that

$$\left(K_{n+1} - w(m^*(e_n))^{1-1/\sigma}(1-\beta^\sigma)^{-1/\sigma}\frac{1}{1-1/\sigma}\right)e_{n+1}^{1-1/\sigma} \geq \left(K_n - w(m^*(e_n))^{1-1/\sigma}(1-\beta^\sigma)^{-1/\sigma}\frac{1}{1-1/\sigma}\right)e_n^{1-1/\sigma} \quad (50)$$

and (48) and (49) imply that

$$\left(K_{n-1} - w(m^*(e_n))^{1-1/\sigma}(1-\beta^\sigma)^{-1/\sigma}\frac{1}{1-1/\sigma}\right)e_{n-1}^{1-1/\sigma} \geq \left(K_n - w(m^*(e_n))^{1-1/\sigma}(1-\beta^\sigma)^{-1/\sigma}\frac{1}{1-1/\sigma}\right)e_n^{1-1/\sigma} \quad (51)$$

Note that it cannot be that

$$\lim_{n\to\infty}\left\{K_n - w(m^*(e_n))^{1-1/\sigma}(1-\beta^\sigma)^{-1/\sigma}\frac{1}{1-1/\sigma}\right\} = 0, \quad (52)$$

since if this were the case, then given the indifference condition, it would violate (48) since $\upsilon > 0$. Since (52) cannot hold, then (50) and (51) taking into account that K_n converges imply that

$$\lim_{n\to\infty}\left(e_n^{1-1/\sigma} - e_{n+1}^{1-1/\sigma}\right) \geq 0 \text{ and } \lim_{n\to\infty}\left(e_n^{1-1/\sigma} - e_{n-1}^{1-1/\sigma}\right) \geq 0,$$

which given (3) implies that if either $\sigma < 1$ or $\sigma > 1$, then $\lim_{n\to\infty} e_{n+1} = \lim_{n\to\infty}(e_n - x_n) = e_n$, so that $\lim_{n\to\infty}(x_n/e_n) = 0$, but this violates (41). This establishes that it is not possible for $\Pr\{f_t = 1|f_{t-1} = 0\} > 0$ for $t \geq 1$ in an equilibrium in which war continues occurring forever with positive probability, and this completes the proof of the first part of the proposition.■

Given the first part of the proposition, this implies that either $f_t = 0$ $\forall t$ or $f_0 = 1$. If $f_t = 0$ $\forall t$ then $\mu_t = 1$ $\forall t$ and (38) together with (12) implies that $u'(x_{t+1}) = (1/\beta)u'(x_t)$. If $f_0 = 1$, then the solution to (4) implies that $u'(x_{t+1}) = (1/\beta)u'(x_t)$. ■

Proof of Proposition 3

By the arguments of Lemma 1 and Proposition 2, an equilibrium exists and it must involve either permanent peace or war at date 0. In this proof we characterize the unique equilibrium for four cases: (i) $\sigma > 1$ and $e_0 < \widehat{e}$, (ii) $\sigma > 1$ and $e_0 > \widehat{e}$, (iii) $\sigma < 1$ and $\lim_{m \to \overline{m}} w(m) < \widehat{w}$, and (iv) $\sigma < 1$ and $\lim_{m \to \overline{m}} w(m) > \widehat{w}$. In the first two cases, $\lim_{e \to 0} F(e) \geq 0$ so that war does not take place if there is zero oil endowment, and we show that this implies that the arguments of parts 2 and 3 of Lemma 1 can be used to construct the equilibrium. In the last two cases, $u(0) = -\infty$ so that the characterization of the equilibrium at the zero endowment point depends on whether $\lim_{e \to 0} F(e)$ exceeds or is below 0, and we show that this depends on $\lim_{m \to \overline{m}} w(m)$, the gains from war under maximal armament.

Part 1. We first define $\widehat{e}$ for the $\sigma > 1$ case and $\widehat{w}$ for the $\sigma < 1$ case. We show that country A strictly prefers permanent peace to immediate war if e is below $\widehat{e}$, and vice versa. Moreover, we show that $\widehat{w}$ is between 0 and 1. Formally, if $\sigma > 1$, define $\widehat{e}$ as follows. If $F(e) > 0$ $\forall e$, $\widehat{e} = \infty$. If $F(e) \leq 0$ for some e, then define $\widehat{e} > 0$ as in part 3 of the proof of Lemma 1. We can show that $F(e) > 0$ $\forall e < \widehat{e}$ and $F(e) < 0$ $\forall e > \widehat{e}$. This is because $F(0) = \upsilon > 0$ and $F'(e)$ can be written as

$$F'(e) = (1 - \beta^{\sigma})^{-1/\sigma} e^{1-1/\sigma} \left(1/\sigma - [w(m^*(e))]^{1-1/\sigma}\right). \tag{53}$$

Since $\sigma > 1$, $F'(e) > (<)\, 0$ if $(1/\sigma)^{1/(1-1/\sigma)} > (<)\, w(m^*(e))$. To see how the value of $w(m^*(e))$ relates to the value of e, the first-order condition which characterizes $m^*(e)$ is

$$(1 - \beta^{\sigma})^{-1/\sigma} e^{1-1/\sigma} = \frac{l'(m^*(e))}{[w(m^*(e))]^{-1/\sigma} w'(m^*(e))}, \tag{54}$$

and implicit differentiation of (54) implies that $m^{*\prime}(e) > 0$ so that $w(m^*(e))$ rises in e. From (53), this implies that there exists some $\widetilde{e} \geq 0$ such that $F'(e) > (<)\, 0$ if $e < (>)\, \widetilde{e}$, which implies $F'(\widehat{e}) < 0$ and that $\widehat{e}$ is unique. If $\sigma < 1$, define $\widehat{w} = (1/\sigma)^{1/(1-1/\sigma)} \in (0,1)$.

Part 2. Consider the first case with $\sigma > 1$ and $e_0 < \widehat{e}$, which from part 1, implies that $F(e) > 0$ $\forall e$ so that permanent peace is strictly preferred to immediate war for all feasible endowment levels. From part 2 of Lemma 1, one can construct an equilibrium in pure strategies which features permanent peace. We prove by contradiction that this equilibrium in pure strategies is unique. Our proof relies on the non-existence of an MPCE from date 1 onward in the event of war at date 0. Formally, suppose instead that there is immediate war at date 0, which is the only other possibility. If this is the case, one must consider the implied sequence $\left\{e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right\}_{t=0}^{\infty}$ and continuation equilibrium in the absence of war. Letting μ_t represent the probability of peace at date t under the government's strategy conditional on $f_{t-1} = 0$, we consider the only three possible scenarios: $\mu_1 = 0$, $\mu_1 = 1$, and $\mu_1 \in (0,1)$. We we can show that in neither of these scenarios can there be a continuation equilibrium which is an MPCE. If $\mu_1 = 0$, (10) together with (3), (10), (12), and (13) implies that $x_0^{S*} = e_0$ so that $e_1^* = 0$. However, if $e_1^* = 0$, then country A's optimal strategy is to choose $\mu_1 = 1$, yielding a contradiction. Suppose if instead $\mu_1 = 1$. Propositions 1 and 2 imply that firms and country A expect $\mu_t = 1$ $\forall t \geq 1$. This means that the unique allocations which satisfies (3), (10), (12), and (13) is $\left\{e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right\}_{t=0}^{\infty} = \left\{\widetilde{e}_t(e_0), u'(\widetilde{x}_t(e_0)), \widetilde{x}_t(e_0), \widetilde{x}_t(e_0)\right\}_{t=0}^{\infty}$, so that country

A receives $U^C(e_0)$ by choosing $f_t = 0$ and $m_t = 0$. However, since $F(e_0) > 0$, this implies that country A can deviate to $m_0 = 0$ and $f_0 = 0$ and be made strictly better off relative to $m_0 = m^*(e_0)$ and $f_0 = 1$. Finally, suppose if instead $\mu_1 \in (0,1)$, then this implies that country A is indifferent between war and peace at $t = 1$. By Propositions 1 and 2, it must again be the case that firms and country A expect $\mu_t = 1\ \forall t \geq 2$. This implies that the unique sequence of allocations which satisfies (3), (10), (12), and (13) starting from $t = 1$ onward admits $\left(e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right) = \left(\widetilde{e}_{t-1}(e_1), u'(\widetilde{x}_{t-1}(e_1)), \widetilde{x}_{t-1}(e_1), \widetilde{x}_{t-1}(e_1)\right)\ \forall t \geq 1$. This implies that country A receives $U^C(e_1)$ at date 1 by deviating to $\mu_1 = 1$. Since $F(e_1) > 0$ since $e_1 \in (0, \widehat{e})$, this implies that such a deviation strictly increases welfare. Therefore, permanent peace is the unique equilibrium starting from any $e_t < \widehat{e}$. Therefore, if $e_0 < \widehat{e}$, the equilibrium is in pure strategies since country A can be confined to choosing $\Pr\{f_t = 1\} = \{0,1\}$ and it never chooses $\Pr\{f_t = 1\} \in (0,1)$ for any $e_t \leq e_0$.

Part 3. Now consider the case with $\sigma > 1$ and $e_0 > \widehat{e}$, which from part 1, implies that $F(e) < 0\ \forall e > \widehat{e}$, so that war is strictly preferred to permanent for some high feasible endowment levels. From part 3 of Lemma 1, one can construct an equilibrium in mixed strategies which features immediate war. We prove by contradiction that this mixed strategy equilibrium is unique. Suppose instead that the equilibrium entailed permanent peace with $\left\{e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right\}_{t=0}^{\infty} = \left\{\widetilde{e}_t(e_0), u'(\widetilde{x}_t(e_0)), \widetilde{x}_t(e_0), \widetilde{x}_t(e_0)\right\}_{t=0}^{\infty}$ and country A receiving $U^C(e_0)$. Because $F(e_0) < 0$, this implies that country A can be made strictly better off by deviating to $m_0 = m^*(e_0)$ and $f_0 = 1$. Therefore, immediate war is the unique equilibrium starting from any $e_t > \widehat{e}$. To show that the equilibrium is in mixed strategies, suppose that country A were confined the pure strategies for all $e_0 > \widehat{e}$ and choose $e_0 > \widehat{e}/\beta$. We can show that an equilibrium does not exist. Consider the strategy of the government at e_1^* conditional on $f_0 = 0$. If $f_1 = 1$, (10) together with (3), (10), (12), and (13) implies that $x_0^{S*} = e_0$ so that $e_1^* = 0$. However, if $e_1^* = 0$, the country A chooses $f_1 = 0$, yielding a contradiction. If instead $f_1 = 0$ at e_1^*, then Proposition 2 implies that $f_t = 0$ at $e_t^*\ \forall t \geq 1$. This implies that the unique sequence of allocations which satisfies (3), (10), (12), and (13) is $\left\{e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right\}_{t=0}^{\infty} = \left\{\widetilde{e}_t(e_0), u'(\widetilde{x}_t(e_0)), \widetilde{x}_t(e_0), \widetilde{x}_t(e_0)\right\}_{t=0}^{\infty}$ so that country A receives $U^C(\widetilde{e}_1(e_0))$ at date 1 starting from e_1^*. However, because $e_1^* = \widetilde{e}_1(e_0) = \beta^{\sigma} e_0 > \widehat{e}$, $F(e_1^*) < 0$ so that country A can make itself strictly better off by deviating to $m_1 = m^*(e_1)$ and $f_1 = 1$. Since an equilibrium in pure strategies does not exist, and since an equilibrium in mixed strategies exists by Lemma 1, the unique equilibrium is in mixed strategies.■

Part 4. Now consider the case with $\sigma < 1$ and $\lim_{m \to \overline{m}} w(m) < \widehat{w}$. We argue that this implies that $F(e) > 0\ \forall e$ which means that the arguments of part 2 of Lemma 1 can be used to construct a pure strategy equilibrium with permanent peace. We prove by contradiction that this is the unique equilibrium. Suppose instead that there is immediate war. Formally, let us assume and later prove that $F(e) > 0\ \forall e$. In this case, the same arguments as in part 2 imply that an equilibrium with immediate war does not exist and that the equilibrium is in pure strategies. In order to prove that $F(e) > 0\ \forall e$, we show that $F'(e) < 0\ \forall e$ and that $\lim_{e \to \infty} F(e) > 0$. We can establish that $F'(e) < 0\ \forall e$ from (53), since this is true given that

$w(m^*(e)) < \widehat{w}\ \forall e$. To establish that $\lim_{e\to\infty} F(e) > 0$, consider first the value of $\lim_{e\to\infty} m^*(e)$. Suppose that $\lim_{e\to\infty} m^*(e) = \underline{m} > 0$. This would violate (54) since the left hand side of (54) would converge to 0 whereas the right hand side of (54) would converge to a positive number. Therefore, $\lim_{e\to\infty} m^*(e) = 0$ which implies that

$$\lim_{e\to\infty} \left(V\left(w\left(m^*(e)\right)e\right) - l\left(m^*(e)\right) - \upsilon\right) = -\upsilon,$$

so that $\lim_{e\to\infty} F(e) = \upsilon > 0$. This establishes that $F(e) > 0\ \forall e$ which verifies our assumption.

Part 5. Now consider the case with $\sigma < 1$ and $\lim_{m\to\overline{m}} w(m) > \widehat{w}$. We argue that this implies that $\lim_{e\to 0} F(e) = -\infty$, which from the arguments of part 1 of Lemma 1 implies that we can construct a pure strategy equilibrium with immediate war. We prove by contradiction that this pure strategy equilibrium is unique. Formally, let us assume and later prove that $\lim_{e\to 0} F(e) = -\infty$ which means that $f_t = 1$ if $e_t = 0$. By the continuity of $F(e)$, it also implies that that there exists some $\widehat{e}$ such that $F(e) < 0\ \forall e \leq \widehat{e}$. Suppose that there is an equilibrium with permanent peace so that $\left(e_t^*, p_t^*, x_t^{S*}, x_t^{A*}\right) = \left(\widetilde{e}_t(e_0), u'\left(\widetilde{x}_t(e_0)\right), \widetilde{x}_t(e_0), \widetilde{x}_t(e_0)\right)\ \forall t \geq 0$. This implies that $e_t^* = \beta^{\sigma t} e_0 < \widehat{e}$ for some t which is sufficiently large and that country A receives $U^C(e_t)$ starting from e_t. Country A can deviate to $m_t = m^*(e_t)$ and $f_t = 1$ and be made strictly better off since $F(e_t) < 0$, so that permanent peace is not an equilibrium and the unique equilibrium is immediate war. To see that the equilibrium is in pure strategies let us confine country A to choosing $\Pr\{f_t = 1\} = \{0, 1\}$, and construct an equilibrium in pure strategies as in part 1 of Lemma 1. Finally, let us prove that $\lim_{e\to 0} F(e) = -\infty$. We first consider $\lim_{e\to 0} m^*(e)$. Suppose that $\lim_{e\to 0} m^*(e) = \overline{m}' < \overline{m}$. This would violate (54) since the left hand side of (54) approaches ∞ as e approaches 0, whereas the right hand side of (54) approaches $l'(\overline{m}') / \left[\left[w(\overline{m}')\right]^{-1/\sigma} w'(\overline{m}')\right] < \infty$, yielding a contradiction. Since $\lim_{e\to 0} m^*(e) = \overline{m}$, it follows that $\lim_{e\to 0} w(m^*(e)) > \widehat{w}$. Now consider $\lim_{e\to 0} F(e)$ which satisfies:

$$\lim_{e\to 0} F(e) = \lim_{e\to 0} \left(V\left(w\left(m^*(e)\right)e\right) - l\left(m^*(e)\right) - \upsilon\right) \left(\frac{U^C(e)}{V\left(w\left(m^*(e)\right)\right) - l\left(m^*(e)\right)} - 1\right). \tag{55}$$

The first term on the right hand side of (55) converges to $-\infty$ and we can show that the second term approaches a positive number. This is because by L'Hopital's rule and (54) which defines $m^*(e)$.

$$\begin{aligned}\lim_{e\to 0} \frac{U^C(e)}{V\left(w\left(m^*(e)\right)e\right) - l\left(m^*(e)\right) - \upsilon} &= \frac{dU^C(e)/de}{d\left(V\left(w\left(m^*(e)\right)e\right) - l\left(m^*(e)\right) - \upsilon\right)/de} \\ &= \frac{1/\sigma}{\lim_{m\to\overline{m}} \left[w(m)\right]^{1-1/\sigma}} > 1.\end{aligned}$$

Therefore, $\lim_{e\to 0} F(e) = -\infty$, which verifies our assumption.■

7.2 Proofs from Section 4

Proof of Lemma 2

Following the discussion in the text, the existence of an MPME is guaranteed by the existence of a function $U_S(e_t)$ which satisfies (22). Substitute (3) and (21) into (19) which binds to achieve

$$-c_t = G(e_{t+1}, e_t) = u(e_t - e_{t+1}) + \beta (V(w(m^*(e_{t+1}))e_{t+1}) - l(m^*(e_{t+1}))) - V(w(m^*(e_t))). \tag{56}$$

Taking the above relationship into account, substitute (18) into (22) so as to write $U_S(e_t)$ as:

$$U_S(e_t) = \max_{f_t=\{0,1\}, e_{t+1}\in[0,e_t]} \{(1-f_t)[G(e_{t+1}, e_t) + \beta U_S(e_{t+1})] + f_t\psi\} \tag{57}$$

By construction, $U_S(e_t) \in [\psi, V(\overline{e})]$ for some arbitrarily large $\overline{e} > e_t$ so that it is bounded. Define the operator $J : \mathcal{B}([0,\bar{e}]) \to \mathcal{B}([0,\bar{e}])$ as

$$(JU_S)(e_t) = \max_{f_t=\{0,1\}, e_{t+1}\in[0,e_t]} \{(1-f_t)[G(e_{t+1}, e_t) + \beta U_S(e_{t+1})] + f_t\psi\},$$

where $\mathcal{B}([0,\bar{e}])$ is the space of bounded above functions which map $[0,\bar{e}]$ into $\mathbb{R}$ for some arbitrarily large $\bar{e}$. $\mathcal{B}([0,\bar{e}])$ is equipped with its sup norm is a metric space. J maps $\mathcal{B}([0,\bar{e}])$ into itself and is a contraction by Blackwell's Sufficiency Conditions. Therefore by the Contraction Mapping Theorem, a unique $U_S(e_t)$ exists. ■

Proof of Proposition 4

This is proved by a variational argument which considers a specific perturbation on the solution in which starting from e_t, the choice of e_{t+1} is increased by $\epsilon \gtreqless 0$ arbitrarily small, where this increase is accommodated by a decrease in x_t by ϵ and an increase in x_{t+1} by ϵ.

Let e^*_{t+1} denote the optimal choice of e_{t+1} starting from e_t. Equation (57) implies that since $f_{t+1} = 0$, then $f_t = 0$ and

$$U_S(e_t) = u\left(e_t - e^*_{t+1}\right) + \beta\left[V\left(w\left(m^*\left(e^*_{t+1}\right)\right)e^*_{t+1}\right) - l\left(m^*\left(e^*_{t+1}\right)\right)\right] - V(w(m^*(e_t))e_t) + \beta U_S\left(e^*_{t+1}\right). \tag{58}$$

Since $f_{t+1} = 0$, (58) also holds replacing e_t with e^*_{t+1} and e^*_{t+1} with e^*_{t+2}, where e^*_{t+2} denotes the optimal choice of e_{t+2} starting from e^*_{t+1}.

Optimality requires that the solution at e_t weakly dominates the choice of $e^*_{t+1}+\epsilon$ for $\epsilon \gtreqless 0$. Let $x^*_t = e_t - e^*_{t+1}$ and let $x^*_{t+1} = e^*_{t+1} - e^*_{t+2}$. Optimality of the choice of e^*_{t+1} implies

$$\begin{aligned} u(x^*_t) + \beta\left[V\left(w\left(m^*\left(e^*_{t+1}\right)\right)e^*_{t+1}\right) - l\left(m^*\left(e^*_{t+1}\right)\right)\right] + \beta U_S\left(e^*_{t+1}\right) \geq \\ u(x^*_t - \epsilon) + \beta\left[V\left(w\left(m^*\left(e^*_{t+1}+\epsilon\right)\right)\left(e^*_{t+1}+\epsilon\right)\right) - l\left(m^*\left(e^*_{t+1}+\epsilon\right)\right)\right] + \beta U_S\left(e^*_{t+1}+\epsilon\right). \end{aligned} \tag{59}$$

Since starting from $e^*_{t+1}+\epsilon$ country S can always choose policy e^*_{t+2} associated with e^*_{t+1} together with $f_t = 0$, this implies that

$$\begin{aligned} U_S\left(e^*_{t+1}+\epsilon\right) \quad &\geq \quad U_S\left(e^*_{t+1}\right) + u\left(x^*_{t+1}+\epsilon\right) - u\left(x^*_{t+1}\right) \\ &\quad +V\left(w\left(m^*\left(e^*_{t+1}\right)\right)e^*_{t+1}\right) - V\left(w\left(m^*\left(e^*_{t+1}+\epsilon\right)\right)\left(e^*_{t+1}+\epsilon\right)\right). \end{aligned} \tag{60}$$

Combining (59) with (60) we achieve:

$$\begin{aligned} &\left[u\left(x^*_t\right) - u\left(x^*_t - \epsilon\right)\right] - \beta\left[u\left(x^*_{t+1}+\epsilon\right) - u\left(x^*_{t+1}\right)\right] \\ &\quad +\beta\left[l\left(m^*\left(e^*_{t+1}+\epsilon\right)\right) - l\left(m^*\left(e^*_{t+1}\right)\right)\right] \geq 0. \end{aligned} \tag{61}$$

Divide both sides of (61) by $\epsilon \gtrless 0$ and take the limit as ϵ approaches 0. This yields:

$$u'\left(x_t\right) - \beta u'\left(x_{t+1}\right) + \beta l'\left(m^*\left(e_{t+1}\right)\right) m^{*\prime}\left(e_{t+1}\right) = 0. \tag{62}$$

Since $l'(\cdot) > 0$, (62) implies that $u'\left(x_{t+1}\right) > (<)\,(1/\beta)\, u'\left(x_t\right)$ if $m^{*\prime}\left(e_{t+1}\right) > (<)\, 0$. ■

Proof of Proposition 5

Given (8), the first-order condition which defines $m^*\left(e_t\right)$ is

$$l'\left(m_t\right) = V'\left(w\left(m_t\right)e_t\right) w'\left(m_t\right) e_t. \tag{63}$$

Given the solution to (4), the envelope condition implies that

$$V'\left(w\left(m_t\right)e_t\right) = \beta^k u'\left(x_{t+k}\right) \;\; \forall k \geq 0. \tag{64}$$

Substitution of (64) into (63) followed by implicit differentiation yields

$$\left(\frac{l''\left(m_t\right)}{\beta^k u''\left(x_{t+k}\right) w'\left(m_t\right) e_t} - \frac{u'\left(x_{t+k}\right) w''\left(m_t\right)}{u''\left(x_{t+k}\right) w'\left(m_t\right)}\right)\frac{dm_t}{de_t} = \frac{dx_{t+k}}{de_t} + \frac{u'\left(x_{t+k}\right)}{u''\left(x_{t+k}\right) e_t}. \tag{65}$$

The resource constraint implies

$$\sum_{k=0}^{\infty}\frac{dx_{t+k}}{de_t} = w\left(m_t\right) + w'\left(m_t\right)\frac{dm_t}{de_t}.$$

Taking the sum of (65) $\forall k \geq 0$ and substitution into the above equation yields

$$\frac{dm_t}{de_t} = \frac{w\left(m_t\right)\left(1 + \sum_{k=0}^{\infty}\frac{u'\left(x_{t+k}\right)}{u''\left(x_{t+k}\right)x_{t+k}}\frac{x_{t+k}}{w\left(m_t\right)e_t}\right)}{\sum_{k=0}^{\infty}\left(\frac{l''\left(m_t\right)}{\beta^k u''\left(x_{t+k}\right) w'\left(m_t\right) e_t} - \frac{u'\left(x_{t+k}\right) w''\left(m_t\right)}{u''\left(x_{t+k}\right) w'\left(m_t\right)}\right) - w'\left(m_t\right)}. \tag{66}$$

Since the denominator in (66) is negative, the term is positive if and only if the numerator is negative. If it is the case that $-u'(x_{t+k})/u''(x_{t+k})\,x_{t+k} > 1\ \forall x_{t+k}$ then the numerator is negative since $\sum_{k=0}^{\infty} \frac{x_{t+k}}{w(m_t)\,e_t} = 1$, and the opposite holds if $-u'(x_{t+k})/u''(x_{t+k})\,x_{t+k} < 1$ $\forall x_{t+k}$. ■

Proof of Proposition 6

Part 1. Suppose that (26) holds. We can prove by contradiction that the equilibrium cannot involve war occur starting from any e_t. Suppose that there was war at e_t and suppose that country S deviates by offering country A $x_t^o = (1-\beta^\sigma)\,w(m^*(e_t))\,e_t$ and

$$-c_t^o = u(x_t^o) + \beta\left(V\left(w\left(m^*\left(e_t - x_t^o\right)\right)\left(e_t - x_t^o\right)\right) - l\left(m^*\left(\left(e_t - x_t^o\right)\right)\right)\right) - V\left(w\left(m^*\left(e_t\right)\right)\right), \quad (67)$$

so that country A accepts the offer. Such an offer provides country S with a payoff which is strictly greater than ψ so that it dominates making an offer which is rejected and leads to war. More specifically, since $m^*(\cdot)$ is a decreasing function which is bounded from above by $\overline{m}$, it is the case that $w(m^*(e_t - x_t^o)) \geq w(m^*(e_t))$ and $-l(m^*((e_t - x_t^o))) \geq -l(\overline{m})$. Moreover, it is also the case that $U_S(e_t - x_t^o) \geq \psi$, since starting from $e_t - x_t^o$, country S can always offer country A $\{0,0\}$ which must be rejected and leads to war since accepting the offer provides $-\infty$ utility to country A. Therefore, country S's welfare from making this offer exceeds

$$u(x_t^o) + \beta\left(V\left(w\left(m^*\left(e_t\right)\right)\left(w\left(m^*\left(e_t\right)\right)e_t - x_t^o\right)\right) - l\left(\overline{m}\right)\right) - V\left(w\left(m^*\left(e_t\right)\right)\right) + \beta\psi,$$

By definition, $u(x_t^o) + \beta V\left(w\left(m^*\left(e_t\right)\right)\left(w\left(m^*\left(e_t\right)\right)e_t - x_t^o\right)\right) = V\left(w\left(m^*\left(e_t\right)\right)\right)$, so that the above term reduces to $-\beta l(\overline{m}) + \beta\psi$, which exceeds ψ if (26) holds. This proves the first part of the proposition.■

Part 2. If preferences satisfy (17) for $\sigma < 1$ and $w(\overline{m}) > (1/\sigma)^{1/(1-1/\sigma)}$, then war occurs with probability 1 in the MPCE by Proposition 3. Suppose that (26) also holds. Then war is avoided in the MPME by step 1. Since country S's welfare exceeds ψ in the MPME whereas it is equal to ψ in the MPCE, country S is better off in the MPME. This proves the second part of the proposition.■

Part 3. Suppose preferences satisfy (17) for $\sigma < 1$ and (27) holds. We can construct an MPME in which war occurs starting from all $e_t \leq e_0$. Suppose that starting from all e_t, country S offers $\{0,0\}$ and country A rejects the offer and declares war. It is clear that country A's strategy given the offer is optimal since accepting the offer yields a welfare of $-\infty$. To show that country S's strategy is optimal, consider the best offer which country S could make at t conditional on it being accepted given that war occurs with certainty at $t+1$. Such an offer satisfies (67) so that it is a solution to the below program:

$$\max_{e_{t+1}} \left\{u\left(e_t - e_{t+1}\right) + \beta V\left(w\left(m^*\left(e_{t+1}\right)\right)e_{t+1}\right) - \beta l\left(m^*\left(e_{t+1}\right)\right) - V\left(w\left(m^*\left(e_{t+1}\right)\right)e_{t+1}\right) + \beta\psi\right\}.$$

Since $w(m^*(e_{t+1})) \leq 1$, $u(e_t - e_{t+1}) + \beta V(w(m^*(e_{t+1}))e_{t+1}) \leq V(e_t)$. Moreover, since $m^*(\cdot)$ is a decreasing function, $-\beta l(m^*(e_{t+1})) \leq -\beta l(m^*(e_t))$. Therefore, the value of the above objective cannot exceed

$$V(e_t) - \beta l(m^*(e_t)) - V(w(m^*(e_t))e_t) + \beta\psi = V(e_t)(1 - w(m^*(e_t))) - \beta l(m^*(e_t)) + \beta\psi,$$

which is strictly below ψ by (27), which means that letting war occur is optimal. This proves the third part of the proposition.■

Part 4. Suppose that preferences satisfy (17) for $\sigma < 1$ and $w(\overline{m}) < (1/\sigma)^{1/(1-1/\sigma)}$. Country S receives ψ in the MPME. War does not occur in the MPCE by Proposition 3 and country S's welfare equals

$$\sum_{t=0}^{\infty} \beta^t u'(\widetilde{x}_t(e))\widetilde{x}_t(e) = (1 - 1/\sigma)V(e_0)$$

in the MPCE. Since $(1 - 1/\sigma)V(e_0) > 0 > \psi$, country S is strictly better off in the MPCE.■

7.3 Proofs from Section 5

Proof of Proposition 7

Part 1. Given the discussion in the text, country S's program can be written as:

$$U_S(e_t) = \max_{x_t \geq 0, c_t} \{-c_t - l(m_S^*(e_t)) + \beta U_S(e_{t+1})\} \text{ s.t. (3) and}$$
$$u(x_t) + c_t + \beta[V(w(m_A^*(e_{t+1}), m_S^*(e_{t+1}))e_{t+1}) - l(m_A^*(e_{t+1}))] = V(w(m_A^*(e_t), m_S^*(e_t))e_t).$$

Now consider the solution given that $f_t = f_{t+1} = 0$. Let e_{t+1}^* denotes the implied optimal choice of e_{t+1} starting from e_t so that

$$\begin{aligned} U_S(e_t) &= u\left(e_t - e_{t+1}^*\right) - l(m_S^*(e_t)) + \beta\left[V\left(w\left(m^*\left(e_{t+1}^*\right)\right)e_{t+1}^*\right) - l\left(m^*\left(e_{t+1}^*\right)\right)\right] \\ &\quad -V(w(m^*(e_t))e_t) + \beta U_S\left(e_{t+1}^*\right). \end{aligned} \tag{68}$$

Follow the same perturbation arguments as in the proof of Proposition 4. This yields:

$$\begin{aligned} &[u(x_t^*) - u(x_t^* - \epsilon)] - \beta\left[u\left(x_{t+1}^* + \epsilon\right) - u\left(x_{t+1}^*\right)\right] \\ &+\beta\left[l\left(m_A^*\left(e_{t+1}^* + \epsilon\right)\right) - l\left(m_A^*\left(e_{t+1}^*\right)\right) + l\left(m_S^*\left(e_{t+1}^* + \epsilon\right)\right) - l\left(m_S^*\left(e_{t+1}^*\right)\right)\right] \geq 0. \end{aligned} \tag{69}$$

Divide both sides of (69) by $\epsilon \gtrless 0$ and take the limit as ϵ approaches 0. This yields:

$$u'(x_t) - \beta u'(x_{t+1}) + \beta l'(m_A^*(e_{t+1}))m_A^{*\prime}(e_{t+1}) + \beta l'(m_S^*(e_{t+1}))m_S^{*\prime}(e_{t+1}) = 0. \tag{70}$$

Since $l'(\cdot) > 0$, (70) implies that $u'(x_{t+1}) > (<)(1/\beta)u'(x_t)$ if $m_A^{*\prime}(e_{t+1}) > (<) 0$ $m_S^{*\prime}(e_{t+1}) > (<) 0$. ■

Part 2. Analogous arguments to those of Proposition 5 imply that $\widetilde{m}_A^*(e_t, m_{St})$ and $\widetilde{m}_S^*(e_t, m_{At})$ increase (decrease) in e_t if $-u'(x)/(xu''(x)) > (<)\, 1$ for all x. As such, application of Acemoglu and Jensen (2009) imply that $m_A^*(e_t)$ and $m_S^*(e_t)$ increase (decrease) in e_t if $-u'(x)/(xu''(x)) > (<)\, 1$ for all x. ■

Proof of Proposition 8

Part 1. Define

$$\widehat{V}_i(e_t) = V\left(w\left(m_i^*(e_t), \{m_j^*(e_t)\}_{j=1,j\neq i}^N\right) e_t\right)$$

Given the discussion in the text, country S's program can be written as:

$$U_S(e_t) = \max_{\{x_{it}\geq 0, c_{it}\}_{i=1}^N} \left\{-\sum_{i=1}^N c_{it} + \beta U_S(e_{t+1})\right\} \text{ s.t. (30) and}$$
$$u(x_{it}) + c_{it} + \beta\left(\widehat{V}_i(e_{t+1}) \quad - l(m_i^*(e_{t+1}))\right) = \widetilde{V}_i(e_t) \;\; \forall i$$

Now consider the solution given that $f_t = f_{t+1} = 0$. Let x_{it}^* and e_{t+1}^* denotes the implied optimal choice of e_{t+1} starting from e_t so that

$$U_S(e_t) = \sum_{i=1}^N \left(u(x_{it}^*) + \beta\left[\widehat{V}_i(e_{t+1}^*) - l\left(m^*(e_{t+1}^*)\right)\right] - \widetilde{V}_i(e_t)\right) + \beta U_S(e_{t+1}^*). \tag{71}$$

Since $f_{t+1} = 0$, (71) also holds replacing e_t with e_{t+1}^* and e_{t+1}^* with e_{t+2}^*, where e_{t+2}^* denotes the optimal choice of e_{t+2} starting from e_{t+1}^*. Optimality requires that the solution at e_t weakly dominates the choice of $e_{t+1}^* + \epsilon$ for $\epsilon \gtrless 0$ where this is achieved by reducing x_{it}^* by ϵ. Optimality of the choice of e_{t+1}^* implies

$$u(x_{it}^*) + \beta\sum_{j=1}^N \left[\widehat{V}_j(e_{t+1}^*) - l\left(m_j^*(e_{t+1}^*)\right)\right] + \beta U_S(e_{t+1}^*) \geq \tag{72}$$
$$u(x_{it}^* - \epsilon) + \beta\sum_{j=1}^N \left[\widehat{V}_j(e_{t+1}^* + \epsilon) - l\left(m_j^*(e_{t+1}^* + \epsilon)\right)\right] + \beta U_S(e_{t+1}^* + \epsilon).$$

Since starting from $e_{t+1}^* + \epsilon$ country S can always choose policy e_{t+2}^* associated with e_{t+1}^* so that x_{it+1}^* is increased by ϵ this implies that

$$\begin{aligned} U_S(e_{t+1}^* + \epsilon) \;\; &\geq \;\; U_S(e_{t+1}^*) + u(x_{it+1}^* + \epsilon) - u(x_{it+1}^*) \\ &\quad + \sum_{j=1}^N \left[\widehat{V}_j(e_{t+1}^*) - \widetilde{V}_j(e_{t+1}^* + \epsilon)\right]. \end{aligned} \tag{73}$$

Combining (72) with (73) we achieve:

$$\begin{aligned}&\left[u\left(x_{it}^{*}\right)-u\left(x_{it}^{*}-\epsilon\right)\right]-\beta\left[u\left(x_{it+1}^{*}+\epsilon\right)-u\left(x_{it+1}^{*}\right)\right] \\ &+\sum_{j=1}^{N}\beta\left[l\left(m_{j}^{*}\left(e_{t+1}^{*}+\epsilon\right)\right)-l\left(m_{j}^{*}\left(e_{t+1}^{*}\right)\right)\right]\geq 0.\end{aligned} \tag{74}$$

Divide both sides of (74) by $\epsilon \gtrless 0$ and take the limit as ϵ approaches 0. This yields:

$$u'\left(x_{it}\right)-\beta u'\left(x_{it+1}\right)+\sum_{j=1}^{N}\beta l'\left(m_{j}^{*}\left(e_{t+1}\right)\right)m_{j}^{*\prime}\left(e_{t+1}\right)=0. \tag{75}$$

Since $l'(\cdot)>0$, (62) implies that $u'\left(x_{it+1}\right)>(<)\left(1/\beta\right)u'\left(x_{it}\right)$ if $m_{j}^{*\prime}\left(e_{t+1}\right)>(<)\,0\ \forall j$. ■

Part 2. Analogous arguments to those of Proposition 5 imply that $\widetilde{m}_{i}^{*}\left(e_{t},\mathbf{m}_{-it}\right)$ increases (decrease) in e_t if $-u'(x)/\left(xu''(x)\right)>(<)\,1$ for all x. As such, application of Acemoglu and Jensen (2009) imply that $m_{i}^{*}\left(e_{t}\right)$ increase (decrease) in e_t $\forall i$ if $-u'(x)/\left(xu''(x)\right)>(<)\,1$ for all x. ■

Proof of Proposition 9

The solution to (4) implies that given $w_i(\cdot)$

$$V\left(w_{i}(\cdot)\,Ne_{t}\right)=\frac{\left(1-\beta^{\sigma}\right)^{-1/\sigma}}{1-1/\sigma}w_{i}(\cdot)^{1-1/\sigma}N^{1-1/\sigma}e_{t}^{1-1/\sigma}-\frac{1}{\left(1-\beta^{\sigma}\right)\left(1-1/\sigma\right)},$$

which together with the fact that $m_{i}^{*}\left(e_{t}\right)$ is the same across countries implies (32). One can thus write (75) as:

$$\left(1-1/\sigma\right)\left(N-1\right)\left(1-\beta^{\sigma}\right)^{-1/\sigma}e_{t+1}^{-1/\sigma}=x_{it+1}^{-1/\sigma}-\frac{1}{\beta}x_{it}^{-1/\sigma}. \tag{76}$$

One can conjecture in this environment that

$$x_{it+1}^{-1/\sigma}=\frac{1}{\rho}x_{it}^{-1/\sigma}$$

for some constant ρ which may be above or below β. Under this conjecture, it must be that $x_{it}=\left(1-\rho^{\sigma}\right)e_{t}$ and $e_{t+1}=\rho^{\sigma}e_{t}$, which means that (76) can be rewritten as:

$$\left(1-1/\sigma\right)\left(N-1\right)\left(1-\beta^{\sigma}\right)^{-1/\sigma}=\left(1-\frac{\rho}{\beta}\right)\left(1-\rho^{\sigma}\right)^{-1/\sigma}, \tag{77}$$

which verifies that our conjecture of a constant growth rate in the marginal rate of substitution holds. (77) provides an implicit function which characterizes the value of ρ. Note that the derivative of the right hand side of (77) has the same sign as:

$$-\frac{1}{\beta}+\left(\frac{1}{\rho}-\frac{1}{\beta}\right)\rho^{\sigma} \tag{78}$$

which must be negative. This is because if $\sigma<1$, then $\rho>\beta$ so that (78) is negative and if $\sigma>1$, then $\rho<\beta$ and (78) cannot be greater than $-1/\beta+1-\rho/\beta<0$. It follows that if $\sigma<1$, the left hand side of (77) declines as N rises, so that ρ rises as N rises. Alternatively, if $\sigma>1$, the the left hand side of (77) rises as N rises, so that ρ declines as N rises, which completes the argument. ■

Proof of Proposition 10

Analogous arguments as in the proof of Proposition 4 imply that $m_t=m^*(e_t)$, that

$$U_A(e_t)=\widetilde{V}(e_t),$$

and that country S's optimal offer must satisfy:

$$\begin{gathered}U_S(e_t)=\max_{x_t\geq 0,c_t}\left\{-c_t+\beta U_S(e_{t+1})\right\}\text{ s.t. (3) and}\\ u(x_t,c_t,-m^*(e_t))+\beta\widetilde{V}(e_{t+1})=\widetilde{V}(e_t).\end{gathered}$$

Let e^*_{t+1} denote the implied optimal value of e_{t+1} starting from e_t, and let e^*_{t+2} denote the implied optimal value of e_{t+2} starting from e^*_{t+1}. Let $\widetilde{c}_t(\epsilon)$ and $\widetilde{c}_{t+1}(\epsilon)$, respectively, solve:

$$u\left(e_t-e^*_{t+1}-\epsilon,\widetilde{c}_t(\epsilon),-m^*(e_t)\right)+\beta\widetilde{V}\left(e^*_{t+1}+\epsilon\right)=\widetilde{V}(e_t)\text{ and} \tag{79}$$

$$u\left(e^*_{t+1}-e^*_{t+2}+\epsilon,\widetilde{c}_{t+1}(\epsilon),-m^*\left(e^*_{t+1}+\epsilon\right)\right)+\beta\widetilde{V}\left(e^*_{t+2}\right)=\widetilde{V}\left(e^*_{t+1}+\epsilon\right) \tag{80}$$

for $\epsilon\gtreqless 0$. Note that by implicit differentiation:

$$\begin{aligned}\widetilde{c}'_t(0)&=\frac{u_x(x_t,c_t,-m_t)-\beta\widetilde{V}'(e_{t+1})}{u_c(x_t,c_t,-m_t)}\\ \widetilde{c}'_{t+1}(0)&=\frac{-u_x(x_{t+1},c_{t+1},-m_{t+1})+u_m(x_{t+1},c_{t+1},-m_{t+1})\,m^{*\prime}(e_{t+1})+\widetilde{V}'(e_{t+1})}{u_c(x_{t+1},c_{t+1},-m_{t+1})}\end{aligned}$$

Optimality requires that

$$\begin{aligned}\widetilde{c}_t(0)+\beta U_S\left(e^*_{t+1}\right)&\geq\widetilde{c}_t(\epsilon)+\beta U_S\left(e^*_{t+1}+\epsilon\right)\\ &\geq\widetilde{c}_t(\epsilon)+\beta\left(\widetilde{c}_{t+1}(\epsilon)-\widetilde{c}_{t+1}(0)+U_S\left(e^*_{t+1}\right)\right)\end{aligned}$$

which implies that

$$\widetilde{c}_t(0)-\widetilde{c}_t(\epsilon)\geq\beta\left(\widetilde{c}_{t+1}(\epsilon)-\widetilde{c}_{t+1}(0)\right). \tag{81}$$

Divide both sides of (81) by $\epsilon \gtrless 0$ and take the limit as ϵ approaches 0 so as to achieve:

$$-\widetilde{c}_t'(0) = \beta \widetilde{c}_{t+1}'(\epsilon),$$

which by substitution yields:

$$\begin{aligned}\frac{u_x(x_{t+1}, c_{t+1}, -m_{t+1})}{u_c(x_{t+1}, c_{t+1}, -m_{t+1})} &= \frac{1}{\beta}\frac{u_x(x_t, c_t, -m_t)}{u_c(x_t, c_t, -m_t)} + \frac{u_m(x_{t+1}, c_{t+1}, -m_{t+1})}{u_c(x_{t+1}, c_{t+1}, -m_{t+1})} m^{*\prime}(e_{t+1}) \\ &\quad + \widetilde{V}'(e_{t+1})\left(\frac{1}{u_c(x_{t+1}, c_{t+1}, -m_{t+1})} - \frac{1}{u_c(x_t, c_t, -m_t)}\right),\end{aligned}$$

which completes the proof since $u_c(\cdot), u_m(\cdot) > 0$. ■

7.4 Monopolistic Environment without Armament

Here we briefly consider the implications of allowing country A to engage in war without the possibility for armament. In particular, suppose that

$$w(m) = \bar{w} \in (0, 1] \text{ for all } m, \tag{82}$$

which implies that country A never invests in armament in equilibrium.

It is then straightforward to see that wars do not occur in any period. This is because country S can always structure offers to country A so as to replicate the outcome of war while making itself better off by avoiding war which costs it ψ.

Formally, if country A attacks country S over any stock of the resource e_t, country A's payoff is $V(\bar{w}e_t)$ and its path of extraction of the resource following the war $\{\tilde{x}_{t+k}(\bar{w}e_t)\}_{k=0}^{\infty}$ is a solution to (4) when $w(m) = \bar{w}$. Note that it satisfies

$$V(\bar{w}e_t) = u(\tilde{x}_t(\bar{w}e_t)) + \beta V(\bar{w}e_t - \tilde{x}_t(\bar{w}e_t)). \tag{83}$$

It is feasible for country S to make offers in equilibrium that replicate the payoff of country A in the event of war. In fact, we can show a stronger statement that country S in any period can make an offer that makes both countries strictly better off than having a war. Consider an offer $\tilde{z}_t = \{\tilde{x}_t(\bar{w}e_t), \epsilon\}$ where $\epsilon \in (0, -(1-\beta)\psi)$. Since the payoff of country A in period $t+1$ is bounded by the payoff from attacking country S, $V(\bar{w}(e_t - \tilde{x}_t(\bar{w}e_t)))$, its payoff in period t from accepting offer $\tilde{z}_t$ satisfies

$$\begin{aligned} u(\tilde{x}_t(\bar{w}e_t)) + \epsilon + \beta U_A(e_t - \tilde{x}_t(\bar{w}e_t)) &> u(\tilde{x}_t(\bar{w}e_t)) + \beta V(\bar{w}e_t - \tilde{x}_t(\bar{w}e_t)) \\ &= V(\bar{w}e_t)\end{aligned}$$

where the last line uses (83). This means country A is made strictly better off accepting this alternative offer.

Similarly, the payoff of country S in period $t+1$ is bounded by the payoff from being attacked ψ, since country S can always make an offer which is rejected.[16] Therefore, country S's payoff following the acceptance of the offer is

$$-\epsilon + \beta U_S(e_t - \tilde{x}_t(\bar{w}e_t)) \geq -\epsilon + \beta\psi.$$

Since $-\epsilon + \beta\psi > \psi$, country S is made strictly better off so that war cannot be an equilibrium with any endowment e_t.

Since wars are never an equilibrium, country S makes an offer z_t to extract the maximum surplus from country A subject to avoiding war. We can then show that such an offer always satisfies the Hotelling rule. Formally, country S's maximization problem is

$$U_S(e_t) = \max_{x_t \geq 0, c_t} \{-c_t + \beta U_S(e_{t+1})\} \tag{85}$$

subject to (3),

$$u(x_t) + c_t + \beta U_A(e_{t+1}) \geq V(\bar{w}e_t). \tag{86}$$

With the same argument as in the text, the participation constraint is given by (86) and this constraint must bind; if it did not, country S could strictly improve its payoff by offering a lower value of c_t to country A. Therefore, in this case, $U_A(e_t) = V(\bar{w}e_t)$ for all e_t so that country A is indifferent between attacking and not attacking country S in every period. Therefore, the maximization problem of country S can be written as a maximization of (85) subject to (3), and

$$u(x_t) + c_t + \beta V(\bar{w}e_{t+1}) \geq V(\bar{w}e_t).$$

The first-order conditions to this problem establishes that x_t must satisfy Hotelling rule (16).[17]

It is optimal for country S to equalize country S's marginal rate of substitution over x to the marginal rate of transformation since this is the most efficient means of extracting payments from country A. As an illustration of this intuition, suppose that $\beta u'(x_{t+1}) > u'(x_t)$. If country S extracts ϵ units of resources less in period t and $\epsilon > 0$ more in period $t+1$, holding everything fixed, it changes payoff of country A by $(\beta u'(x_{t+1}) - u'(x_t))\epsilon > 0$, which relaxes constraint (86). This allows country S to reduce c_t and hence increase the payments it receives from country A. If instead $\beta u'(x_{t+1}) < u'(x_t)$, then analogous arguments imply that country S could improve its payoff by extracting $\epsilon > 0$ units of resources more in period t and ϵ less in period $t+1$.

[16] Formally, starting from any e_t, country S can offer $\{0, 0\}$, which yields a payoff $\beta U_S(e_t)$ if it does not lead to war and ψ if it leads to war. This implies that

$$U_S(e_t) \geq \min\{\beta U_S(e_t), \psi\} = \psi, \tag{84}$$

where we have used the fact that if it were the case that $\beta U_S(e_t) < \psi < 0$, (84) would imply $U_S(e_t) \geq 0$, yielding a contradiction.

[17] To take the first-order condition one needs to assume that $U_S(e)$ is differentiable. One can prove the same result without assuming differentiability by following the same steps as in the proof of Proposition 4.

We summarize the results of this section in the following proposition:

Proposition 11 *Suppose $w(\cdot)$ satisfies (82). Then in any MPME:*

1. *War never occurs.*
2. *The equilibrium sequence of resource extraction, x_t, satisfies (16) for all t.*

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