Non-ignorable non-response in roll-call data analysis

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Work in progress – Comments welcome

Abstract

Roll-call votes are widely employed to infer the ideological organization of legislatures, even though ideal points derived from roll-call data can be seen as accurate reflections of underlying policy preferences only under stringent assumptions. We explore the consequences of violating one of such assumptions, namely, the ignorability of the process that generates non-response in roll calls. We offer a reminder of the inferential consequences of ignoring certain processes of non-response, a basic estimation framework to model non-response and vote choice concurrently, and a handful of models of specific non-ignorable processes of non-response that can be used as building blocks to study abstentions and absences in roll-call data.

Ideal points estimated from roll-call analyses constitute accurate reflections of the underlying policy preferences of individual legislators only under very stringent circumstances. Under conditions common in legislative settings—strategic voting, log-rolling, non-independence of vote choice within parties, for example—ideal points are at best imperfect reflections of legislators' ideological profiles. The problem of missingness also affects inferences about ideal points derived from roll-call votes. This problem has two different manifestations: First, roll-call votes may be a non-random selection of all votes taken by a voting body. In such instances, we can conceive of roll-call votes as *observed* data and non-recorded votes as missing data. The *selection bias* on inferred ideal points that follows from this manifestation of the missingness problem has been recently documented by Carrubba, Gabel, Murrah, Clough, Montgomery and Schambach (2006) and Carrubba, Gabel and Hug (2008), among others. In this paper, we explore theoretically some of the consequences of the second manifestation of the missingness problem, which occurs when we encounter non-response (i.e., no indication of vote choice) within roll-call data. This missingness problem might occur even if roll-call votes are a random

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sample or constitute the entire population of votes in a legislative body. We are concerned with the following questions: What is lost when we ignore non-response, as is commonly done in Nominate and Bayesian IRT models? How can one build models of the process of non-response in legislative voting? Are these models likely to yield better inferences about ideal points when the non-response mechanism is known a priori?

As a first step in understanding the problem of non-ignorable non-response in roll-call data, we explore these questions in a simulation setting. We motivate our analysis in Section 1 by considering the frequency of abstentions and absences in legislatures across the world and by reviewing previous literature on non-response. We then build our analysis of abstentions by extending the Bayesian MCMC item response theory (IRT) model of Clinton, Jackman and Rivers (2004*b*). In Section 2 we summarize this model and then review the conditions under which the abstention-generating process can be ignored without loss of information. These "ignorability" conditions were explored by Rubin in the context of inference in the face of missing data (Rubin 1976). We are careful to define explicitly the assumptions of *missing at random* (MAR), *missing completely at random* (MCAR) and, above all, parameter *distinctness* (D), a condition that is often forgotten in applied work on missingness. We should note that though Rubin's conditions also apply to likelihood-based inference, we limit our inquiry to Bayesian inference, and particularly to the context of the MCMC IRT model. In Sections 3 and 4 we build different models of abstention based on theoretical accounts of legislative behavior that can be found in the literature. Based on hypothetical data from simulated legislatures with varying degrees of missing values, we compare inferences about ideal points based on our explicit models of abstention against those based on the Bayesian IRT model. We conclude in Section 5 with an outlay of our agenda for future work.

1 Non-response in roll-call data

Empirical studies of legislative behavior often resort to estimates of ideal points of politicians to examine the influence of interest groups on legislators' choices, the dimensionality of the political spectrum, the ideological coherence of political parties, the degree to which party pressure affects voting behavior, or the identity of pivotal and "most extreme" members of a voting body. Analyses that rely on the estimation of ideal points build on the spatial theory of voting (Enelow and Hinich 1984; Hinich and Munger 1992), which assumes that legislators support a bill if the expected utility of voting for it outweights the expected utility of voting against it. Based on this theory—and on a number of assumptions to allow identification of a statistical model—analysts can recover ideal points from typical legislator-by-vote matrices containing 1's and 0's corresponding to observed Ayes and Nays in the record. However, many of these matrices have missing values that correspond to abstentions or absences. Rosas and Shomer (2008) investigate the frequency of non-response in a convenience sample of voting assemblies in sixteen countries across the world, and find the average rate of missingness to be 24.5%, with a maximum of 68.5% on average in the Israeli Knesset between 1999 and 2008. In short, the rate of missing values in actual voting bodies around the world is rather high, probably higher than the rate of missingness that scholars confront when analyzing survey data or other kind of data sets.

There are at least two reasons why we should worry about non-response in roll-call data analysis. The first reason concerns the possibility of using patterns of non-response as data that can itself be predicted based on alternative theories of legislative voting. As Clinton and Meirowitz (2003) suggest, different theories of legislative voting behavior yield a variety of *maintained* and *to be tested* hypotheses that can be translated into constraints on the parameters of the statistical model that is used to estimate ideal points. For example, under certain models of "competing principals" we might believe that the policy preferences of legislators, in interaction with their parties' stances, may lead them to strategically abstain rather than express their true preference on some crucial votes. In this case, analysts might be interested in building and estimating a full model of both the "observed" voting record and the pattern of non-responses in a roll-call matrix. A model of this sort with good purchase in fitting the pattern of non-response as well as the observed pattern of Ayes and Nays might provide evidence to decide among competing theories of voting behavior. In this regard, our first contribution is that we derive a variety of models that capture mechanisms of abstention commonly found in the literature. These models can be used as building blocks in future research that purports to arbitrate among competing theories of legislative behavior.

We are mostly concerned with the second reason, namely, the potential impact of missingness on inferences about ideal points themselves. That is, even if one is not intrinsically interested in understanding the process that generates abstentions or in arbitrating among competing implications of legislative theories, there are circumstances under which missing data may affect the quality of our inferences concerning ideal points. This problem was pressed by Clinton, Jackman and Rivers (2004*a*) in their analysis of the *National Journal*'s "most liberal" US Senator rank-orders during the 2004 Presidential election. Among author cautionary remarks about the use of roll-call data to infer preferences, the authors suggest that the voting record that won John Kerry the monicker of most liberal senator was probably not observed at random, given that he was at the time a presidential candidate on the campaign trail that could only afford to vote on carefully selected bills in the Senate. As Clinton, Jackman and Rivers (2004*a*, 5) put it, "the roll calls that do draw candidates back to Washington to cast votes are not a random subset of roll calls, but are on issues where their votes might have utmost importance for procedural reasons." Consequently, it is likely that Kerry's vote preferences were systematically different for bills he attended than for bills he missed.[1]

Applied work on roll-call analysis proceeds under the assumption that missing values are inconsequential for the purpose of inferring ideal points. This assumption is reasonable when the process that generates non-response is entirely random, as when legislators miss a series of votes because they fall ill or the researcher loses a subsample of votes to computer failure. In fact, many instances of non-response in the legislatures referenced above should be chalked off to "official absences", which are presumably the consequence of random events unrelated to the voting process. Even then, it is illuminating to see that absences and other forms of non-response tend to be higher in legislatures where explicit abstentions are not recorded. The most dramatic example in this regard comes from the New Zealand Parliament, which did not allow the possibility of abstention until after 1995; consequently, in the voting record corresponding to 1990–1994 there are no explicit abstentions or official absences, but 26.3 legislator/vote cells

[1] Clinton, Jackman and Rivers (2004*a*) reports the non-response rates of Senators on the presidential campaign trail during 2004: Kerry, 60%; Lieberman, 40%; Edwards, 35%. Jackman (2008) show the non-response rates on the 99 key votes that the *National Journal* used in its 2007 vote ratings for Senators that campaigned in primaries during the latest presidential election: McCain, 55%; Obama, 35%; Clinton, 20%. As Clinton, Jackman and Rivers (2004*a*) note, we already witness much uncertainty in nailing down the ideological positions of extremists, but this uncertainty is only exacerbated by a short voting record. Furthermore, if the process that generates these high rates of non-response among Senators in the presidential campaign trail is not-random, inferences about their ideal points will also be biased (see below).

are still empty.

Furthermore, explicit abstentions—i.e., instances in which a legislator affirms the choice neither to vote in favor nor against a bill—make up a significant portion of instances of non-response in many legislatures. Because explicit abstention is a purposeful choice, it is less obvious that the pattern of missingness they give rise to is entirely unrelated to the voting process. In many institutional settings, there are reasons to believe that abstentions, and even physical absences, may not be random, but driven by some underlying characteristic of the voting process. For example, Barack Obama's voting record during his tenure as representative in the Illinois Senate often comes up in journalist accounts of willful abstention. In fact, Obama's non-response rate in that voting body was not extraordinarily high ($\approx 5\%$), but it included a relatively large number of instances in which he was "present but did not vote". According to some reports, non-response in these instances was not random, but either corresponded to directives from the Democratic Party leadership in the State Senate or to instances in which voting (either way) would have alienated at least one important constituency.[2]

Scholarly work on abstentions in roll-call voting in the United States House and Senate explores the reasons why legislators might choose not to vote, and commonly concludes that abstentions are intentional and strategic acts (Forgette and Sala 1999; Kromer 2005; Rothenberg and Sanders 1999; 2000; 2002; Zupan 1991). These efforts are generally not followed by attempts to decide whether the implied patterns of missingness should be considered ignorable. Instead, in line with the argument that abstentions may be strategic, these analyses directly attempt to model participation and vote choice decisions jointly, for example using two-stage estimation or simultaneous equations models. These arguments fall neatly in the first category we identified above, namely, within research that purports to find empirical support for theoretical accounts of legislatures by testing implications about abstentions. Thus, Cohen and Noll (1991) examine vote choice and abstention in the US Congress on a series of roll-call votes on the Clinch River Reactor, assuming that voting behavior is geared towards improving a legislator's re-election chances. The authors find evidence that indifferent legislators are more likely to participate

[2] See, *inter alia*, the following stories: Raymond Hernandez and Christopher Drew, "It is not just 'Ayes' and 'Nays': Obama's votes in the Illinois echo", *New York Times*, December 20, 2007; Ginger A. Otis, "A Barack-and-forth voter", *New York Post*, February 17, 2008; Michael Dobbs, "Candidate Watch: Obama's voting record on abortion", *Washington Post*, February 6, 2008.

in a vote when the vote is close and the outcome is uncertain, whereas they are less likely to participate when the outcome can be clearly predicted. Poole and Rosenthal (1997) derive non-response from spatial voting motivations by assuming that abstentions are a function of the chance that a legislator will be pivotal on any given vote. Rothenberg and Sanders (1999; 2002) also argue that abstention and vote choice should be considered as jointly determined. In their analysis of the 73$^{\text{rd}}$, 89$^{\text{th}}$ and 104$^{\text{th}}$ US Congresses they find support for the hypothesis that abstention is not random, especially in the 104$^{\text{th}}$ Congress. In a different institutional context and based on different sets of assumptions about legislative behavior, Noury (2004) uses roll-call data from the third and fourth European Parliaments (1989-1999) and finds that closeness is an important determinant of abstention, that being in the majority decreases the likelihood for abstaining, and that increases in fixed costs (such as vote timing and the physical place where sessions take place) increase abstentions. Except for the latter effect, the inescapable conclusion is that abstentions are indeed purposeful acts.

We also find in previous literature attempts to recover ideal point estimates unaffected by missingness in roll-call data. These attempts can be catalogued as efforts to measure the policy preferences of legislators by diminishing bias introduced by non-random non-response. Desposato (2006) discusses several realistic scenarios in which abstentions should be considered strategic and proposes a model that takes into consideration the strategic nature of abstention. In order to correct for bias introduced by non-random abstentions, he estimates a mixture model with vote choice as well as an underlying latent variable which determines whether a legislator registers a vote or not. His approach, however, does not consider that models of non-response should be tailored to the assumed mechanism that generates abstentions. Rosas and Shomer (2008) explore the problem of strategic non-response in the Argentine Congress and the Israeli Knesset. They purport to counter the bias introduced by strategic non-response through simultaneous estimation of ideal points and legislator-specific abstention propensities, which are also allowed to predict the observed pattern of Ayes and Nays. This effort was aided by theoretical knowledge about the institutional milieu that leads different Argentine legislators and Israeli MKs to abstain in some votes. However, even based on solid theoretical knowledge about the conditions under which legislators prefer to abstain, it is not entirely clear how much is gained by explicitly modeling non-response or, conversely, how much is lost by assuming that patterns of non-response are random. It is mostly the lack of a clear yardstick against which to compare improvement that hinders our efforts to deal with non-response in roll-call votes. We believe this problem can best be remedied through analysis of simulated datasets, where we have control over the mechanism that generates both vote choice and non-response.

Political scientists have drawn inspiration from the literature on educational and psychological testing, which faces obstacles similar to those produced by missingness in roll-call data and has produced a voluminous literature on modeling non-ignorable non-response in IRT models. As Mislevy and Wu (1996) suggest, examinees often fail to respond fully to all items in a test for reasons that may or may not be intended by the administrator and that may or may not be correlated with the examinees' ability. Perhaps the most common mechanism of non-ignorable non-response in educational testing occurs when the ability parameter also drives the propensity not to respond (DeMars 2002). Mislevy and Wu (1996) suggest that item-response theory models need to be tailored to the characteristics of the specific mechanism that is suspected to produce non-response. As far as we know, the idea of using a latent variable approach to analyze response propensities and ability propensities was pioneered in psychological testing by O'Muircheartaigh and Moustaki (1999). The models developed by O'Muircheartaigh and Moustaki (1999) and expanded by Holman and Glas (2005) are useful in that they extract information about the underlying abilities of individuals from their very patterns of non-response. In the context of educational and psychological testing, these models can be extended to make non-response propensities a function of observed covariates. Thus, Moustaki and Knott (2000) build models of non-response conditional on observed covariates; estimated propensities of non-response can then be used to weight items and respondents so as to correct for the effect of missingness (see also Pimentel (2005)).[3] However, the ability to condition non-response on observed characteristics of individuals is one advantage that is seldom granted to political scientists analyzing roll-call votes.[4]

In the next sections we consider the problem of non-ignorable non-response within the

[3] We use a variant of this approach, without observed covariates, to study Argentine and Israeli legislative bodies in Rosas and Shomer (2008).

[4] An alternative to concurrent modeling is a two-step procedure in which imputed values are produced for non-responses in the first step, followed by estimation of an IRT model in the second step (Sheng and Carrière 2005). Though this approach turns the model estimation stage into a simpler problem, it still requires theoretically-guided statistical adjustment to impute values in the first stage.

Bayesian item-response theory (IRT) model of Clinton, Jackman and Rivers (2004*b*). We start by summarizing theoretical knowledge about the conditions under which missingness mechanisms can be ignored. We make the case that the "missing at random" assumption that underlies the Bayesian IRT model is not sufficient to ignore the process that generates missing data, even when the assumption holds. We also develop different models corresponding to different processes of abstention. One conclusion of our research is that it is unlikely that a single, general-purpose model of abstention can be constructed. Instead, models need to be tailored to the specific non-response generating mechanism that is suspected to be at play.

2 The IRT model and non-ignorable non-response

Scholars commonly resort to the Bayesian item-response theory model formalized by Clinton, Jackman and Rivers (2004*b*) to study various aspects of legislative politics. This statistical model follows naturally from theoretical micro-foundations about the behavior of legislators that confront an up-or-down vote on an item that changes the statu quo in a policy dimension. Following Clinton, Jackman and Rivers (henceforth CJR), we consider x_i, ζ_j, $\psi_j \in \mathbb{R}^1$ as the ideological position of legislator $i \in \{1 \ldots I\}$, and the Aye and Nay locations, respectively, of bill $j \in \{1 \ldots J\}$ in one-dimensional ideological space. Legislators derive utility from the spatial distance between their own ideal point x_i and the locations ζ_j and ψ_j of bills, i.e., $U_i(\text{aye}_j) = -(x_i - \zeta_j)^2 + \eta_{ij}$ and $U_i(\text{nay}_j) = -(x_i - \psi_j)^2 + \nu_{ij}$, where η_{ij} and ν_{ij} are spherical disturbances. Consistent with the spatial theory of voting, the utility differential $y^*_{ij} = U_i(\zeta_j) - U_i(\psi_j)$ determines whether a legislator will vote for or against proposal j; we assume that the legislator will vote Aye if $y^*_{ij} > 0$ and will vote Nay otherwise.

For the purpose of statistical estimation, the probability that legislator i will vote in favor of proposal j can be parameterized as $\Pr(y_{ij} = 1) = \Pr(y^*_{ij} \geq 0) = \Phi(\beta_j x_i + \alpha_j)$.[5] Inference about parameters $\boldsymbol{\theta} = \{\boldsymbol{\alpha}, \boldsymbol{\beta}, \mathbf{x}\}$ is based on data $\mathbf{Y}$, an $n \times m$ matrix of 1s and 0s corresponding to Ayes and Nays. Based on the CJR specification, the likelihood function for this model appears

[5] In this parameterization, $\beta_j = 2(\zeta_j - \psi_j)/\sigma_j$, $\alpha_j = (\psi_j^2 + \zeta_j^2)/\sigma_j$, σ_j is the standard deviation of the difference of the disturbance terms (i.e., $\sigma_j^2 = \text{var}(\eta_{ij} - \nu_{ij})$), and Φ is the CDF of the standard normal distribution (a probit link).

in Equation 1:

$$\mathcal{L}(\boldsymbol{\alpha}, \boldsymbol{\beta}, \mathbf{x}|\mathbf{Y}) = \prod_{j=1}^{J}\prod_{i=1}^{I}\Phi(\beta_j x_i - \alpha_j)^{y_{ij}}[1 - \Phi(\beta_j x_i - \alpha_j)]^{1-y_{ij}} \tag{1}$$

In practice, maximum-likelihood estimates of $\boldsymbol{\theta}$ are difficult to derive due to the multiplicity of parameters in need of estimation (the number of parameters to estimate increases with the size of data). Conditional on stipulating prior distributions on these parameters, however, Bayesian estimates are easily derived through Markov-Chain Monte Carlo algorithms.

In many applications, some elements of $\mathbf{Y}$ are missing because of abstentions or absences. The MCMC algorithm used to estimate this model handles missing values easily by sampling model parameters based on data (artificially completed using initial guesses for missing values), then sampling missing data "on the fly" based on estimated model parameters. This process is iterated a large number of times; as a by-product of parameter estimation, the MCMC algorithm returns a distribution of imputed values for each missing cell in $\mathbf{Y}$.[6] In fact, missing values simply constitute another set of parameters to be estimated in the Bayesian framework. In practical terms, high rates of missing values mean that information about underlying parameters $\boldsymbol{\theta}$ is less plentiful. This has the effect of increasing uncertainty about parameter estimates, but the procedure still guarantees reasonable estimates of ideal points $\boldsymbol{\theta}$ *as long as we can assume that the non-response mechanism is ignorable.* Whenever non-response cannot be assumed to be ignorable, Bayesian (and likelihood-based) methods, including the CJR model, can lead to biased and/or inefficient inferences about parameters of interest.[7] Thus, understanding the potential benefits of modeling missingness mechanisms in roll-call data means understanding the conditions under which missingness processes should be deemed non-ignorable.

Rubin (1976) derives the conditions under which the missingness process can be disregarded in a model based on observational data. We summarize Rubin's conditions for ignorability within the context of abstentions and absences in roll-call analysis. We assume that roll-calls

[6]Multiple imputation through data augmentation is the mechanism that yields imputed values for the missing data (Albert and Chib 1993; Tanner and Wong 1987).

[7]Aside from the assumption of ignorability, the CJR IRT model assumes local independence and strict monotonicity. Indeed, the one-dimensional specialization of the CJR model is part of a family of models known by the acronym SMURFLI(2), since they are based on the following assumptions: **s**trict **m**onotonicity, **u**nidimensional **r**esponse **f**unction, **l**ocal **i**ndependence, and **two** possible responses (Aye or Nay in our case) (Mislevy and Wu 1996).

are realizations $\mathbf{z}$ of a random variable $\mathbf{Z} = (\mathbf{Z}_1, \ldots, \mathbf{Z}_I)$, where $\mathbf{Z}_i = (Z_{i1}, \ldots, Z_{iJ})'$ comprises the sequence of *vote choices* of legislator i on items 1 through m, *regardless of whether these vote choices are observed or not.* We further assume that elements of $\mathbf{Z}$ are independent and identically distributed with probability density function $f(\mathbf{Z}|\theta)$, or simply $f_\theta(\mathbf{Z})$, and we seek to make inferences about parameter(s) θ. In the CJR model, θ is a set that includes ideal points and item parameters (i.e., $\mathbf{x}$, $\boldsymbol{\alpha}$, and $\boldsymbol{\beta}$.)

Roll-call votes are seldom observed entirely. In many instances, a subset of legislators will fail to register a vote on a particular item, either because they are absent or because they abstain. Thus, aside from $\mathbf{Z}$, roll-call votes are patterned by a second random process of non-response produced by variable $\mathbf{M} = (\mathbf{M_1}, \ldots, \mathbf{M_I})$, with typical realization $m_{ij} = 1$ if we *observe* legislator i's decision on item j and $m_{ij} = 0$ otherwise. We define the conditional probability that $\mathbf{M}$ takes on values $\mathbf{m} = (\mathbf{m}_1, \ldots, \mathbf{m}_I)$ given that $\mathbf{Z}$ has taken values $\mathbf{z} = (\mathbf{z}_1, \ldots, \mathbf{z}_I)$ as $g(\mathbf{M} = \mathbf{m}|\mathbf{Z} = \mathbf{z}, \phi)$, or simply as $g_\phi(\mathbf{M}|\mathbf{Z})$. Again, ϕ is a set that may include legislator-specific and item-specific parameters, and possibly elements of θ. The probability density function g_ϕ captures the process that produces abstentions. Whether non-responses are ignorable or not, the *complete-data likelihood* for a model of votes and abstentions considers both the *vote choice* and the *non-response* processes:

$$\mathcal{L}(\theta, \phi|\mathbf{Z}, \mathbf{M}) = f_\theta(\mathbf{Z})g_\phi(\mathbf{M}|\mathbf{Z}).$$

Similarly, the *complete-data joint posterior distribution* of θ and ϕ is

$$\pi(\theta, \phi|\mathbf{Z}, \mathbf{M}) \propto \underbrace{f_\theta(\mathbf{Z})g_\phi(\mathbf{M}|\mathbf{Z})}_{\text{Likelihood}}\underbrace{p(\boldsymbol{\phi}, \boldsymbol{\theta})}_{\text{Prior}} \tag{2}$$

where we express priors on θ and ϕ explicitly as a joint distribution to admit the possibility that the parameters that drive the missingness process are not "distinct" from the parameters that drive the voting mechanism.[8]

Whenever we encounter abstentions and absences, we observe a subset $\mathbf{Y}_{(1)}$ of $\mathbf{Z}$ (unobserved elements of $\mathbf{Z}$ are $\mathbf{Y}_0$). The default position of many analysts is to assume that non-response

[8]The likelihood itself can be represented as a joint likelihood $h_{\phi,\theta}(\mathbf{M}, \mathbf{Z})$, rather than as the product of a marginal and a conditional distribution.

is ignorable, then proceed to make likelihood-based inferences about θ from the *observed-data likelihood*:

$$\mathcal{L}(\theta, \phi|\mathbf{Y}_{(1)}) = f_\theta(\mathbf{Y}_{(1)}).$$

Maximum-likelihood inferences about θ proceed from the observed-data likelihood by averaging over the (unknown) values of $Y_{(0)}$. Similarly, Bayesian inferences are based on the *observed-data posterior distribution*, which reduces to:

$$\pi(\theta, \phi|\mathbf{Y}_{(1)}) \propto f_\theta(\mathbf{Y}_{(1)})p(\boldsymbol{\theta}) \tag{3}$$

As can be seen from comparing the observed-data and complete-data posterior distributions (Equations 2 and 3), ignoring missing observations in Bayesian inference is tantamount to dropping $g_\phi(\mathbf{M}|\mathbf{Z})$ and $p(\boldsymbol{\phi}, \boldsymbol{\theta})$, i.e., the part of the model that aims to capture the abstention process as well as a priori information on ϕ and on the relation between ϕ and θ. Under the assumption of ignorability that underlies CJR, it is reasonable to make inferences about θ based on the observed-data posterior distribution (Equation 3). If this assumption indeed holds, modeling the process that generates abstentions *will not improve inferences* about the parameters that produce Aye/Nay votes. Another way of saying this is that the abstention-generating mechanism is ignorable for inferences about θ if inferences based on the complete-data posterior in Equation 2 coincide with inferences based on the observed-data posterior in Equation 3 (Lu and Copas 2004).

Following Rubin's theorems, the abstention-generating mechanism is ignorable under Bayesian inference under two sets of conditions:

1. Abstentions are ignorable if (a) the missing data are missing at random (MAR) *and* (b) parameters θ and ϕ are distinct (D).[9,10]

2. Abstentions are ignorable if and only if the probability of the pattern of missingness $\mathbf{M}$

[9]Missing data are MAR if for each value of ϕ, $g_\phi(\mathbf{M}|\mathbf{Z}) = g_\phi(\mathbf{M}|\mathbf{Y}_{(1)})$. In the context of roll-call votes, MAR means that the probability that any given legislator/bill cell will be missing does not depend on the value in the cell (i.e. the legislator's vote choice). Missing data are MCAR if for each value of ϕ, $g_\phi(\mathbf{M}|\mathbf{Z}) = g_\phi(\mathbf{M})$, that is if the probability that any given legislator/bill cell will be missing is constant across bills and legislators.

[10]Parameters θ and ϕ are said to be distinct "if there are no *a priori* ties, via parameter space restrictions or prior distributions" between them. Technically, from Rubin's Definition 3, "[t]he parameter ϕ is distinct from θ if their joint parameter space factorizes into a ϕ-space and a θ-space, and when prior distributions are specified for θ and ϕ, these are independent" (Rubin 1976, 585).

given $\mathbf{Y}_{(1)}$ is the same for all values of θ.[11]

The latter is a necessary and sufficient condition that says that ignorability may still hold even when MAR does not, but this rests on an unlikely balancing condition linking θ to ϕ.[12] Thus, in practice, we require that the non-response process is MAR *and* that the parameters of the non-response and vote choice processes are distinct. In Bayesian inference, distinctness holds when the prior distribution of θ and ϕ can be factored out as $p(\theta)p(\phi)$—i.e., as the product of two independent distributions, with no functional dependencies between θ and ϕ and no restrictions imposed on the parameter space $\phi \times \theta$. Thus, the non-response process in roll-call voting is ignorable if we can assume MAR *and* D.[13]

2.1 Illustration of missingness mechanisms

To illustrate the consequences of basing inferences about parameters on *observed* data under different missingness mechanisms, consider the records of two legislators with ideal points at $x_1 = -0.5$ and $x_2 = 0.5$, who are asked to vote on 500 bills (Aye=1, Nay=0.) We assume that the process that drives vote choice is such that the probability of voting Aye increases on ideology; specifically, $\Pr(Z = 1|x) = \Phi(x)$. The first row (labeled 0) in Table 1 shows a subsample (15 randomly chosen bills) of a realization of this process for the leftist legislator with ideal point x_1. From realized values of $\mathbf{z}$, we can estimate the ideal points of these two individuals through a transformation of their average voting record, i.e, $\widehat{x}_i = \Phi^{-1}(\bar{x}_i)$. For any one particular realization $\mathbf{z}$ of $\mathbf{Z}$, estimated values $\widehat{\mathbf{x}}$ will differ from the true values $\mathbf{x}$ because of randomness in the vote choice process, but Rubin's conditions entail that $\widehat{\mathbf{x}}$ will be an unbiased

[11] In Rubin's words, (3) equals (2) "if and only if $\mathsf{E}_{\phi,\mathbf{Y}_{(0)}}\{g_\phi(\mathbf{M}|\mathbf{Z})|\mathbf{M},\mathbf{Y}_{(1)},\theta\}$ takes on a constant positive value" and if θ and ϕ are distinct (Rubin 1976, 587).

[12] Namely, as $\{g_\phi(\mathbf{M}|\mathbf{Z})|\mathbf{M},\mathbf{Y}_{(1)},\theta\}$ changes under different values of θ, $f_\theta(\mathbf{Y})$ changes correspondingly in such a way that the product $f_\theta g_\phi$ remains unaltered (cf. Mislevy and Wu 1996). Thus, a process of this sort requires that the non-response density g_ϕ itself changes under different values of θ, and that these changes exactly counterbalance the probability that an observation will be missing given vote choice.

[13] Note that MAR neither implies nor is implied by D; these are separate assumptions. In a good segment of the literature on missingness, however, it is customary to use the term MAR loosely to refer both to Rubin's MAR condition and Rubin's D condition. Consider Holman and Glas's definition of MAR: "If $g_\phi(\mathbf{M}|\mathbf{Y}_{(0)},\mathbf{Y}_{(1)})$ does not depend on the unobserved data $\mathbf{Y}_{(0)}$ and the parameter of the missing-data process, $\boldsymbol{\phi}$, is distinct from the parameter $\boldsymbol{\theta}$ of the distribution of $\mathbf{Y}_{(0)}$ and $\mathbf{Y}_{(1)}$, then the data are MAR" (Holman and Glas 2005, p. 1; we employ our notation for consistency). Some other texts define MAR and D separately, but then assume D throughout (cf. Schaffer 1997). It is therefore not uncommon to read that MAR suffices to ignore the abstention-generating mechanism under Bayesian inference; what these texts really mean is that MAR + D suffice to guarantee ignorability.

Table 1: Subsample of 15 votes from a realization of different non-response processes (Aye=1, Nay=0, NA=*)

0	No NAs	1	0	1	1	1	1	1	0	0	0	1	1	1	1	0
1	MCAR	1	0	1	*	*	1	*	*	0	0	1	1	*	1	*
2	MAR + D	*	0	1	1	1	*	1	*	0	0	*	1	1	*	0
3	MAR	1	*	1	*	1	1	1	*	0	0	1	1	1	1	*
4	MNAR	1	0	1	1	*	*	1	0	0	0	*	1	1	*	0
5	MNAR	1	*	1	1	1	1	1	0	0	0	1	1	1	1	*

estimator of $\mathbf{x}$ if all data are observed or if non-response is ignorable.

Consider now five different probabilistic missingness processes. The first process is "missing completely at random" with a probability of non-response by any legislator on any one vote fixed at a constant value. The corresponding subsample of *observed* data $\mathbf{y}_1$ that is generated by this missingness process appears in the row labeled "1" in Table 1. The second process is "missing at random" with parameter distinctness, which in this case means that the probability of non-response is constant across votes cast by the same legislator, but not across legislators, and that the legislator-specific probability of non-response is independent of the legislators' ideal points (#2). The third process is "missing at random" *without* parameter distinctness; thus, the probability of non-response varies across legislators (though still not across votes within legislator) and is proportional to the legislators' ideal points (#3). Note that parameter distinctness does not hold in this case because the ideal points $\mathbf{x}$ inform *both* the vote choice process *and* the non-response process. The fourth process is "not missing at random", since there is a strictly positive probability of non-response if vote choice is Aye, but the probability of non-response is 0 if vote choice is Nay (#4). The final process is also "not missing at random", but here the probability of non-response is positive if vote choice is Aye for the leftist legislator, positive if vote choice is Nay for the rightist legislator, and 0 otherwise.

Table 2 inspects the consequences of ignoring the process of non-response, which entails drawing inferences about $\mathbf{x}$ exclusively from the *observed* data. The numerical cells in Table 2 report the sampling distribution of $\widehat{\mathbf{x}}$ and mean square error (MSE) statistics over 10,000 draws based on the theoretical processes of *vote choice* and *non-response* described in the previous paragraphs. The second column summarizes the known characteristics of the non-response

Table 2: Sampling distribution of $\widehat{\mathbf{x}}$ (mean, standard error, and root mean square error) based on observed data under different non-response processes (10,000 draws)

			$x_1 = -0.5$			$x_2 = 0.5$		
Process	Conditions	Ignorable?	Mean	SE	RMSE	Mean	SE	RMSE
0	No NAs	Yes	−0.501	0.059	0.055	0.501	0.058	0.055
1	MCAR	Yes	−0.502	0.068	0.071	0.501	0.067	0.071
2	MAR + D	Yes	−0.501	0.071	0.071	0.501	0.071	0.071
3	MAR	No	−0.502	0.107	0.105	0.502	0.070	0.071
4	MNAR	No	−0.909	0.071	0.415	0.072	0.069	0.434
5	MNAR	No	−0.910	0.071	0.416	0.909	0.071	0.415

process and the third column displays whether Rubin's non-ignorability conditions hold.

In row 0, where all data happen to be observed, estimates of $\mathbf{x}$ that ignore the missingness process are unbiased and maximally efficient. Modeling the missingness-generating process in this trivial case would yield no payoff, as we would be conditioning parameter estimates on an indicator matrix $\mathbf{M}$ that does not vary. For the two other ignorable mechanisms (#1 and #2), it is obvious that the only effect of non-response is an efficiency loss in estimating $\mathbf{x}$, as can be seen from increases in the standard errors and root MSE statistics of $\widehat{\mathbf{x}}$.

Instead, ignoring non-response when the non-response process is in fact non-ignorable is consequential. For the MAR process without parameter distinctness (#3), we see that the estimate $\widehat{x}_1$ is still unbiased, but that ignoring non-response yields a noticeably larger standard error of $\widehat{x}_1$. This result suggests that modeling non-response under the assumption of non-distinctness will mostly yield improvements in efficiency. Recall that in the MAR + D mechanism that we assumed, the rate of random non-response of the rightist legislator is smaller than the rate of non-response of the leftist legislator, which explains why $\widehat{x}_2$ is a more efficient estimate of x_2 than $\widehat{x}_1$ is of x_1.

Finally, we see from the two MNAR processes #4 and #5 that the main effect of assuming ignorability is very pronounced bias in $\widehat{\mathbf{x}}$. When Aye votes are more likely to be missing (#4), both ideal points are underestimated by about half the distance between x_1 and x_2. When Aye votes are more likely to be missing for the leftist legislator, and Nay votes are more likely to be missing for the rightist legislator (#5), the leftist's ideal point is severely underestimated and the rightist's ideal point is severely overestimated. More problematically, the standard errors of $\widehat{\mathbf{x}}$ remain relatively narrow, underscoring the fact that by ignoring non-response in a "not missing at random" process we may recover a very wrong estimate with rather high certainty. This point is confirmed by the vast increase in the root MSE statistics of these two processes.

Before inspecting in Section 3 the consequences of non-ignorable non-response in more realistic scenarios, we note that none of the five missingness patterns explored in this section are severe enough to change the rank-order of the legislators. The pattern of missingness would need to be more contrived to arrive at the conclusion that the rightist legislator has an ideal point to the left of the leftist legislator. In other words, when we are interested exclusively in recovering the rank order of legislators, any scaling technique will probably deliver usable responses.

3 Non-ignorable abstention mechanisms

In this section, we model three non-ignorable abstention-generating mechanisms, which we label *indifference*, *alienation*, and *competing principals*. The first two mechanisms—indifference and alienation—produce MAR abstentions, but none of these mechanisms satisfy the condition of parameter distinctness. The third mechanism—competing principals—produces abstentions that are not missing at random (MNAR). These models correspond to admittedly simplified accounts of some theoretical mechanisms of non-response found in the literatures on legislative voting and mass electoral behavior (cf. Carey 2007; Feddersen and Pesendorfer 1996; Fiorina 1974). Needless to say, these mechanisms are not an exhaustive catalogue of all processes that can generate non-ignorable abstention. Our goal in modeling these specific mechanisms is to derive complete-data likelihood functions for non-ignorable processes, to suggest an estimation technique for these functions, and to understand if anything is gained over the already quite robust CJR model by fully modeling vote choice and non-response.

We explain the logic behind each abstention-generating mechanism, as well as the rationale for considering them non-ignorable, in the following subsections. For each mechanism, we derive a full model of vote choice *and* abstention, derive the complete-data likelihood, and explain what information can be considered available *a priori*. Then in Section 4 we compare inferences about legislators' ideal points based on the observed- and complete-data posterior distributions.

3.1 Indifference

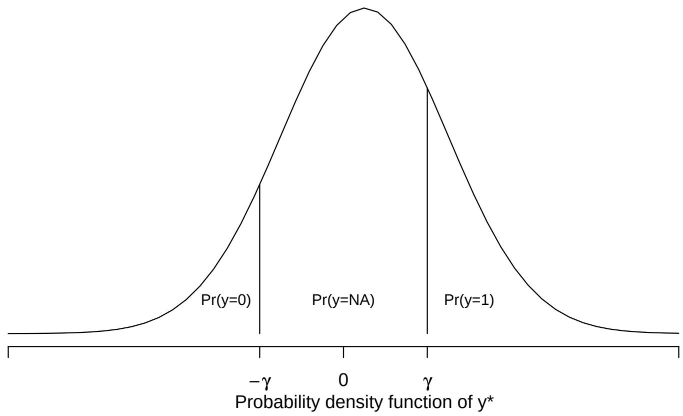
For this mechanism, we posit that legislators may not register a preference if they consider that the utility differential between the Aye and Nay locations of a particular vote is trivial. In other words, this abstention mechanism is one where legislators refrain from voting either when the distance between status quo and bill proposal is minimal or when the distance between status quo and bill proposal is meaningful but their ideological position is more or less equidistant from these two points. We further assume that a legislator abstains when the utility differential y^*_{ij} on any given vote drops below a certain individual-specific tolerance threshold γ_i, and we also stipulate that tolerance thresholds and ideal points are uncorrelated. This latter assumption implies that the abstention process is MAR. To see this, consider that legislators abstain because, *without regard for how they would have voted had they chosen to take the time to consider the options*, (a) it is not worth their time voting on a bill that might change the status quo ever so slightly or (b) they perceive the Aye and Nay positions as consequential, but equally unpalatable.

Consider then what we know about the vote choice and non-response processes under these assumptions. We can assume that legislator i registers a vote on bill j ($m_{ij} = 1$) only if the absolute value of his utility differential is larger than his tolerance threshold, i.e., only if $|y^*_{ij}| > \gamma_i$. As a consequence, we can also derive the conditions that lead to observation of positive votes ($y_{ij} = 1$), negative votes ($y_{ij} = 0$), and missing values ($y_{ij} = *$) in the roll-call matrix:

$$y_{ij} = \begin{cases} 1 & \text{if} \;\; y^*_{ij} \geq \gamma_i \\ * & \text{if} \;\; \gamma_i > y^*_{ij} \geq -\gamma_i \\ 0 & \text{if} \;\; -\gamma_i > y^*_{ij} \end{cases} \tag{4}$$

Legislator i's utility differential on bill j is a random variable. To turn this theoretical abstention mechanism into a statistical model, we derive the probability of observing each event in Equation 4. These probabilities appear in Equation 5, and a graphical representation

Figure 1: Illustration of indifference mechanism of vote and abstention generation



of these events appears in Figure 1.

$$
\begin{aligned}
\Pr(y_{ij} = 1) &= 1 - \Phi(-y^*_{ij} + \gamma_i) \\
\Pr(y_{ij} = *) &= \Phi(-y^*_{ij} + \gamma_i) - \Phi(-y^*_{ij} - \gamma_i) \\
\Pr(y_{ij} = 0) &= \Phi(-y^*_{ij} - \gamma_i)
\end{aligned} \tag{5}
$$

Based on Equation 5, we construct the complete-data likelihood function for the indifference mechanism, using z as an indicator of vote choice and m as an indicator of observed votes:

$$
\begin{aligned}
\mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\phi} | \mathbf{m}, \mathbf{z}) = \prod_{j=1}^{J} \prod_{i=1}^{I} \left[1 - \Phi(-y^*_{ij} + \gamma_i)\right]^{z_{ij} m_{ij}} \times \Phi(-y^*_{ij} - \gamma_i)^{(1 - z_{ij}) m_{ij}} \\
\times \left[\Phi(-y^*_{ij} + \gamma_i) - \Phi(-y^*_{ij} - \gamma_i)\right]^{1 - m_{ij}}
\end{aligned} \tag{6}
$$

Note that the functional form of the indifference model is similar to an ordered probit specification with individual-specific cutpoints.[14] Vote inclination y^*_{ij} is a function of $\boldsymbol{\theta} = \{x_i, \alpha_j, \beta_j\}$ and abstention propensity m^*_{ij} is a function of $\boldsymbol{\phi} = \{x_i, \alpha_j, \beta_j, \gamma_i\}$. Though this non-response mechanism is MAR, the fact that $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$ share common elements means that the assumption of distinctness is not warranted. Since parameters are not distinct, the abstention-generating process is no longer ignorable, and inference should proceed from the complete-data posterior distribution in Equation 6.

[14]The cutpoints are identified through a sum-to-zero constraint.

3.2 Alienation

We now turn to a simplified account of alienation, which is based on arguments in electoral studies that suggest that citizens abstain if they feel alienated from parties' proposed policy positions (Downs 1957; Plane and Gershtenson 2004; Riker and Ordeshook 1968). Following this insight, we model a mechanism where legislators with more extreme ideal points are more likely to abstain on any given bill. Our simplification consists of assuming that the probability of non-response does not depend on the bill parameters.

In this mechanism, abstention rates and ideal points are associated; in fact, the assume mechanism induces a positive correlation between abstention rates and the absolute value of ideal points in our legislatures. However, this mechanism is also MAR because abstentions are not driven by vote choices z_{ij}. In other words, the probability of observing a vote does not depend on the legislator's desired outcome. The logic is easy to convey: Legislators abstain based on individual-specific propensities, *regardless of the direction of their vote had they bothered to register one.*

We proceed as in the previous section to derive the complete-data posterior distribution for this mechanism. Notice first that vote choice can still be captured through the CJR model. Note also that because the abstention process is MAR, conditioning on y^*_{ij} adds no information that we could use to estimate abstention propensities m^*_{ij}. Instead, we can model the abstention propensities of legislators as functions of their ideal points, as shown in Equation 7:

$$\Pr(m_{ij} = 1) = \Pr(m^*_{ij} > 0) = \Phi(\lambda_0 + \lambda_1 x_i - \lambda_2 x_i^2)$$

Because we can assume that y^* and m^* are independent after conditioning on ideal points $\mathbf{x}$, we write the complete-data likelihood function as in Equation 7:

$$\begin{aligned}\mathcal{L}(\boldsymbol{\theta}, \phi|\mathbf{m}, \mathbf{y}) = \prod_{j=1}^{J}\prod_{i=1}^{I}&[\Phi(\beta_j x_i - \alpha_j)\Phi(\lambda_0 + \lambda_1 x_i + \lambda_2 x_i^2)]^{y_{ij} m_{ij}} \\ &\times [(1 - \Phi(\beta_j x_i - \alpha_j))\Phi(\lambda_0 + \lambda_1 x_i + \lambda_2 x_i^2)]^{(1-y_{ij})m_{ij}} \\ &\times [1 - \Phi(\lambda_0 + \lambda_1 x_i + \lambda_2 x_i^2)]^{1-m_{ij}} \qquad (7)\end{aligned}$$

The parameters that characterize the vote inclination and abstention propensity are, respectively, $\theta = \{\boldsymbol{\alpha}, \boldsymbol{\beta}, x\}$ and $\phi = \{x, \boldsymbol{\lambda}\}$, so we cannot assume parameter distinctness. Once we admit that parameters may not be distinct, *the MAR assumption is no longer sufficient to justify ignorability of the non-response mechanism.* Indeed, knowing that a legislator missed a heavy proportion of votes already tells us that her ideal point was more likely to be non-centrist. We incorporate this additional information by including parameter $\mathbf{x}$ in both the vote choice and abstention mechanisms.

3.3 Competing principals

Indifference and alienation are examples of abstention mechanisms that are not ignorable despite the fact that they generate random missingness. This characteristic makes these mechanisms useful yardsticks against which to compare the benefits of explicitly modeling abstentions. However, it would be hard to argue for the relevance of these mechanisms in the analysis of voting bodies such as legislatures or committees. Whereas voters may abstain because of indifference and alienation, politicians and experts presumably have the means and motivation to register a vote even when a bill's Aye and Nay positions are similar or when they are ideologically distant from moderate legislators. As Poole (2005, 90) argues for the US Congress, "most legislators place a high premium on *not* abstaining, because avoiding a vote is often used against them in the next election." This does not mean that abstentions and absences in voting bodies occur completely at random. For our final example, we have chosen an abstention mechanism—"competing principals"—that is theoretically relevant in legislative analysis. Depending on the Aye and Nay locations of a bill, a subset of representatives will be trapped in a competing principals dilemma: their conscience (or district) impels them to vote in some way, but the majority of their party does not. Under these circumstances, these legislators may register an abstention, rather than crossing one of their principals.[15]

[15] Fiorina (1974) argues that legislators that represent heterogeneous constituencies should be more likely to abstain, based on the logic that abstentions are strategically palatable to a legislator torn apart by "competing principals" (see also Carey 2007). In this case, a legislator may want to vote in favor of proposals that she likes, but will prefer to abstain in the understanding that such a vote will anger her constituency. Fiorina also suggests that legislators may avoid paying the opportunity cost of voting whenever they engage in other electorally-related activities, such as devoting time to their constituents (home style). Though these abstentions are also "intentional", they need not violate the MAR assumption if the (unseen) vote choice of the legislator does not drive her decision to be away from the floor to avoid voting on certain bills. The mechanism that we model, needless to say, is much simplified.

We develop a logic of *competing principals* where legislators are mostly free to vote their conscience but might abstain if their vote preference does not coincide with the party line. For this purpose, we use two further pieces of information in our simulated legislatures. First, we consider the party membership of legislators, who can belong to any of two parties, L and R, occupying distinct but overlapping regions of the one-dimensional ideological space. Second, we sampled for each item an official "party vote choice" y_{pj} ($p = L, R$). We think of this as the known position of the party whip or party leader, and we use this information to build a statistical model of competing principals.

In the general version of the competing principals model, we posit that the probability that legislators abstain on any given bill is larger if they disagree with their party's position than if they agree. We thus make the propensity to register a vote (m^*) a function of the legislator's agreement with her party, which is itself a function of the legislator's propensity to vote Aye on bill j and the indicator variable corresponding to the party's vote choice on bill j, as in Equation 8:

$$\text{agree}_{ij} = \begin{cases} 1 & \text{if } (y_{ij} - 0.5)(-1)^{y_{pj}-1} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

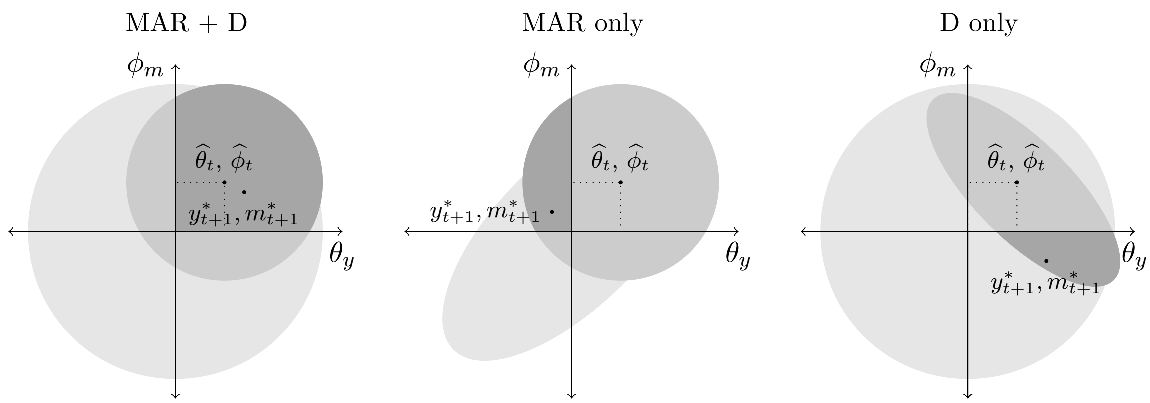
With Equation 8 in place, we parameterize the probability of registering a vote as

$$\Pr(m_{ij} = 1) = \Phi(\delta_1 + \delta_2 \text{agree}_{ij}).$$

The complete-data likelihood for this model follows:

$$\begin{aligned} \mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\phi} | \mathbf{m}, \mathbf{y}) = \prod_{j=1}^{J} \prod_{i=1}^{I} & \Phi(\beta_j x_i - \alpha_j) \Phi(\delta_1 + \delta_2 \cdot \text{agree}_{ij})^{y_{ij} m_{ij}} \\ & \times (1 - \Phi(\beta_j x_i - \alpha_j)) \Phi(\delta_1 + \delta_2 \cdot \text{agree}_{ij})^{(1-y_{ij}) m_{ij}} \\ & \times \left[1 - \Phi(\delta_1 + \delta_2 \cdot \text{agree}_{ij})\right]^{(1-m_{ij})} \end{aligned} \tag{9}$$

Figure 2: Illustration of Rubin's conditions



4 Sampling mechanism and preliminary results

The non-ignorable mechanisms presented in Section 3 require that we estimate the more complex complete-data posterior distributions, which condition inferences on the matrix $\mathbf{M}$ of 1s and 0s that can be built from roll-call sets with incomplete data. For purposes of estimation, we follow the same general strategy in the three cases. As a matter of fact, we incorporate information from both matrices $\mathbf{Y}$ and $\mathbf{M}$ in a model that simultaneously estimates propensities to vote Aye and propensities to respond, an approach based on Holman and Glas (2005). The method proceeds by sampling from a continuous latent variable y^* that represents the propensity to vote in favor of a proposal. In analogous fashion, we sample from a continuous latent variable m^* corresponding to the propensity to register a vote choice. In each iteration, we draw a pair (y^*, m^*) iteratively from a truncated bivariate normal distribution with location parameter $\boldsymbol{\mu} = (\widehat{\theta}_y,\ \widehat{\phi}_m)$ and precision matrix Σ^{-1}. When $y_{ij} = 1$, we truncate this distribution to the set $(y^*_{(+)}, m^*_{(+)})$, where $(+)$ refers to the positive orthant; when $y_{ij} = 0$, we sample from $(y^*_{(-)}, m^*_{(+)})$; finally, when $y_{ij} = \text{NA}$, we sample from $(y^*, m^*_{(-)})$.

Figure 2 displays this method graphically for three different cases: A case where MAR + D can be assumed and abstentions are therefore ignorable; a case where MAR can be assumed, but not D; and a case where D can be assumed, but not MAR. These plots also illustrate graphically the implications of Rubin's conditions. We read each plot as a loose representation of the process of sampling the t^{th} draw of (y^*, m^*) for a single legislator/item pair i, j. The horizontal axis is the locus of $\widehat{\theta}_y$, which is the last iteration of the location parameter of the distribution of latent variable y^*. Conversely, parameter $\widehat{\phi}_m$ (the last iteration of the location parameter of the distribution of latent variable m^*) is represented along the vertical axis. We use the notation θ_y, ϕ_m to remind the reader that these are the parameters that drive the vote choice and abstention mechanisms, respectively, but we identify them with subscripts to suggest that we impose a particular functional form. Thus, $\theta = (\beta, \alpha, x)$, but $\widehat{\theta}_y$ stands specifically for $\beta_j x_i - \alpha_j$ in the IRT model. What varies across models is precisely the functional form of parameters $\widehat{\theta}_y$ and $\widehat{\phi}_m$, as per Equations 6, 7, and 9.

We depict the prior distribution of parameters $\widehat{\theta}_y$ and $\widehat{\phi}_m$ as a light grey contour in each plot. The center of the contour corresponds to the point with highest probability density. Distinct priors are represented as *circular* gray contours, suggesting that the prior distributions of these parameters are independent. *Elliptical* contours convey information about the association among parameters that is captured in the structure of priors. In the second plot, for example, the probability that draw $\widehat{\phi}_m$ takes on a high positive value is higher if $\widehat{\theta}_y$ is also high. To take an example from educational testing, consider all item parameters fixed at values from the previous iteration of the sampling process. If it is known a priori that ability x drives the individual-specific propensity to finish the test γ, then it is known that high values of x are likely to be sampled together with high values of γ and this information should be captured in the structure of prior probabilities. Similarly, information provided by the data likelihood is represented by medium-grey contours. In the rightmost plot, for example, we capture a mechanism where, exploiting another educational testing analogy, respondents are more likely to register a response if the response is in fact wrong than if it is right.

Consider then the data augmentation step performed by the sampling algorithm. If the t^{th} draw from the prior distribution yields the estimate $\widehat{\theta}_y$, $\widehat{\phi}_m$, then point (y^*_{t+1}, m^*_{t+1}) will be sampled from some truncated region of the bivariate normal, represented by the dark grey contour. If $y = 1$, this region will be the darkest region of the contour plot that lies in the first quadrant, as is depicted in the leftmost plot. If $y = 0$, the draw will be from the second quadrant, as in the center plot. Finally, if $y = *$, the draw will be from the third and fourth quadrants, as in the rightmost plot.

We know from Rubin (1976) that patterns of missing and observed votes furnish information that can be used to make inferences about parameters, but we do not know how much better these inferences will be, nor is it clear what the yardstick should be by which we measure improvement. Admittedly, the additional information that can be extracted from conditioning our inferences explicitly on matrix **M** may be minute, and hardly worth the trouble of estimating a complete model. We carry out our analysis through simulations precisely to be able to benchmark our results. This paper reports results on analysis of *one* small simulated assembly; we are currently in the process of estimating all models on a larger set of simulated assemblies, and under alternative assumptions about numbers of legislators and bills. For each of the three abstention mechanisms, we consider three levels of overall non-response—30%, 10%, and 5%—which are common rates of missingness in comparative legislatures. In the simulated assembly, each of 31 legislators is assigned to one of two parties, with the leftist party comprising 18 legislators. Our legislators vote on 100 bills with Aye and Nay positions sampled independently, falling roughly within the center of the distribution of legislators' preferences, but with the restriction that most Aye positions (about 80%) should be to the left of the Nay positions, and thus more likely to be preferred by members of the leftist party. We also sampled "party ideal points", i.e., the policy preferences of the "party whip". For all legislator/item pairs in our simulated set we drew a vote choice—1 or 0 corresponding to Aye or Nay—based on the spatial theoretical reasoning that underlies the CJR model. That is, we assume that legislators prefer a proposal if their utility differential between Aye and Nay bill positions is positive ($y^*_{ij} > 0$).

We compare the models based on complete-data posterior distributions to the CJR model, which assumes MAR non-response and distinct parameters. We expect the CJR model to perform rather well in obtaining appropriate inferences about legislators' ideal points, if only because we built simulated votes based on the very theoretical assumptions that underlie this model. In fact, the observed data in our simulations are set to be extremely informative about bill and legislator positions, even when we use non-ignorable non-response mechanisms to induce heavy rates of missingness. We consider that this very stringent test is a more relevant way to understand what is gained by modeling non-ignorable abstention mechanisms.[16]

[16]Furthermore, the fact that we analyze ideal points in one dimension adds to the expected performance of the CJR model. With one dimension and very informative data, even an eigenvalue decomposition of observed data (plus randomly imputed values) yields reasonable ranks of ideal points. Details about the construction of these legislatures, along with all computer code, can be found online at tadano.wustl.edu/generate-legislatures.htm. Jags and WinBugs code for the non-ignorable non-response models are in the Appendix. Due to the small size of

To hold constant as many details of model specification as possible, all models are identified by stipulating very narrow prior distributions centered on -2 and 2 for the most extremist legislators.[17]

We report in Table 3 statistics to compare inferences about $\mathbf{x}$ from models based on the observed- and the complete-data posterior distributions. Because we have so far finished usable runs for only one legislature, we hasten to add that these data are preliminary and any conclusions based on them are rather tentative. For each mechanism, at each rate of abstention, we display three different statistics that aim to gauge improvement over the CJR model in recovering the parameters used to generate the data and that are similar in spirit to the ones employed in Table 2. The first statistic, Randall's τ, measures the correlation between the rank-order of the original data points and the rank-order recovered by the different models. The second statistic, the average length of the 90% Bayesian credible intervals for ideal points x is a measure of uncertainty surrounding point estimates. Finally, the third statistic is the average root mean square error corresponding to ideal points.

For the legislature we have analyzed so far, we find that adding information found in matrix $\mathbf{M}$ to the estimation of ideal points leads to markedly better results in the MAR processes, i.e., in the indifference and alienation mechanisms. In those cases, the rank-order recovered by the complete-data model is more precise than the rank-order recovered by the observed-data model, the credible intervals for ideal points are narrower when based on the complete-data model, and as a consequence of reduced bias and increased efficiency the average root MSE of ideal points is always smaller in the complete-data model. Finally, we observe that improvements over the CJR model tend to be smaller as the overall rate of missingness decreases in the data. To build intuition about the substantive importance of these improvements, we display in Figure 3 90% credible intervals for estimated ideal points based on complete data (blue series) and observed data (red series) corresponding to the alienation model with a rate of non-response of 5%. Even

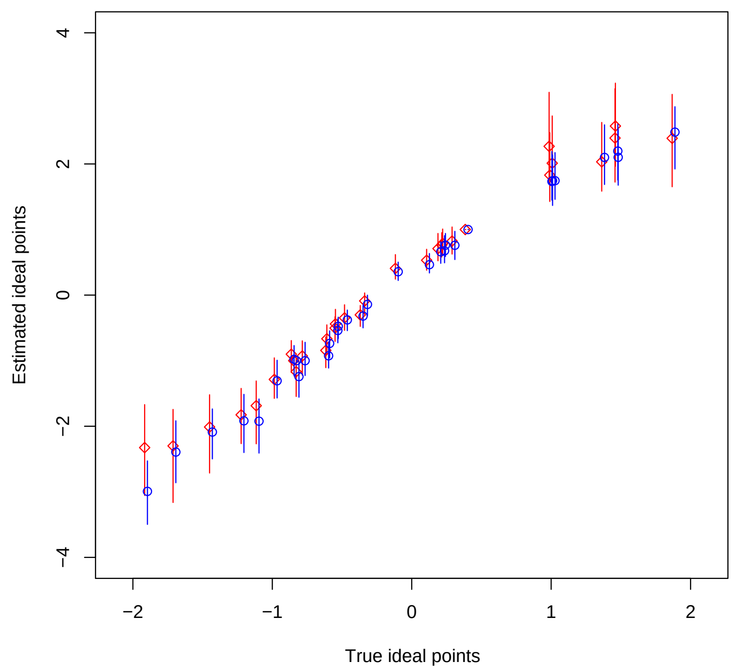
datasets, we typically burn in two chains started at different initial values for 5,000 to 15,000 iterations and base our inferences on 10,000 to 30,000 further iterations. We use the Gelman-Rubin $\widehat{R}$ statistic to assess convergence.

[17] These priors already provide information on the expected order of legislators' positions along the ideological continuum. In the alienation process, we use the legislators that occupy the 25^{th} and 75^{th} percentiles as anchors, fixing their ideal points at -1 and 1, and we stipulate truncated priors on $\boldsymbol{\lambda}$ that are consistent with the a priori knowledge that extremists are more likely to abstain. In the competing principals process we also stipulated truncated priors on $\boldsymbol{\delta}$ consistent with the a priori knowledge that disagreers are more likely to abstain than agreers.

Table 3: Comparison of results of models based on the observed- (O) and complete-data (C) posterior distributions for three different abstention-generating mechanisms at three different average abstention rates (5%, 10%, and 30%)

		Indifference		Extremists		Competing principals	
Rate	Pivots	O	C	O	C	O	C
30%	τ	0.8366	0.9226	0.9140	0.9613	0.8538	0.8581
	Mean 90% CI	1.0660	0.6354	0.7883	0.4099	0.8547	0.8546
	Mean root MSE (x)	1.0137	0.7463	0.3419	0.1736	0.8138	0.8259
10%	τ	0.8731	0.8925	0.9527	0.9613	0.9054	0.9054
	Mean 90% CI	0.7733	0.6494	0.7085	0.5428	0.7598	0.7302
	Mean root MSE (x)	0.7959	0.7136	0.3682	0.3387	0.7595	0.7426
5%	τ	0.8882	0.9140	0.9011	0.9656	0.9183	0.9355
	Mean 90% CI	0.8068	0.7265	0.7921	0.4133	0.7431	0.7629
	Mean root MSE (x)	0.7045	0.6784	0.3488	0.1768	0.7688	0.7369

Figure 3: Differences in inference about ideal points from models based on the observed- and complete-data posterior distributions (blue and red, respectively), for the first abstention mechanism (indifference)



at this comparatively low rate of non-response, it is obvious that there are important gains in terms of efficiency from modeling the missingness process. These gains in efficiency translate into greater ability to recover the correct rank-order of ideal points.

Our results for the competing principals model, which embodies a pattern of non-random non-response, are discouraging. Our comparison statistics show essentially no improvement—or, in the case of the 5% rate of non-response, even slightly worse results than those produced by the model based on observed data. To recapitulate the logic behind this mechanism, we posit that party members in disagreement with the policy position of the party leader might acquiesce to abstain, rather than register a vote choice contrary to the party line. The simulation process by which we generated the legislature was such that the propensity to register a vote on a given bill was higher among "agreers" than among "disagreers", but was non-zero even for the latter group. Furthermore, because we model a two-party legislature and because bill positions are sampled more or less from within the middle of the ideological dimension, on any given vote the number of disagreers of the Left party is about balanced by the number of disagreers on the Right party. These two facts may have combined to yield a pattern of non-response that, while non-random, is not extremely informative.

5 Conclusion

Our research into non-ignorable non-response in roll-call data pursues three goals: First, we seek to clarify the conditions under which non-response processes lead to wrong inferences about the ideological organization of legislatures. We have explored this issue in the context of Bayesian inference, as the Bayesian IRT model of Clinton, Jackman and Rivers (2004*b*) is, among other ideal-point estimation techniques, the most robust to missingness.[18] Second, we have developed a general model specification that exploits the basic idea of data augmentation to estimate the joint posterior distribution of the linear predictors of non-response and vote choice. This general specification is based on work in educational testing and psychometrics developed by Holman and Glas (2005) (see also Glas 2006; Mislevy and Wu 1996; O'Muircheartaigh and Moustaki 1999). In fact, Holman and Glas (2005) propose a simultaneous model of item choice and non-response that models both linear predictors (m^* and y^* in our notation) as two-parameter Rasch models, a specification identical to that of the CJR model. These authors speculate that such general model could in principle detect deviations from random missingness by summarizing the posterior distribution of the correlation between m^* and y^*. Though this procedure may prove advantageous in the study of non-response in educational testing, we are not persuaded about its validity in ideal-point estimation. Contrary to educational testing, where the parameters that drive non-response are often portrayed as linearly related to the parameters that drive ability, in legislative voting we often argue that ideological propensities may be associated with the parameters that drive non-response, but not necessarily in linear fashion.

Our third goal was to derive complete-data likelihood functions for three simple processes of non-ignorable non-response. These were derived from extensions to the spatial theory of voting that underlines the CJR model. Though we strove for simplicity, processes similar to the ones

[18]Indeed, Nominate is ostensibly less robust to problems of missingness in roll-calls, as it provides "correct" inferences about ideal points only under MCAR patterns of non-response.

we portrayed are often invoked as causes of non-response in roll-call votes. We conclude that complete-data models are able to pinpoint the rank-order of ideological positions with greater accuracy than observed-data models, even in legislatures with comparatively small rates of non-response. However, this conclusion holds only for the MAR processes without parameter distinctness that we inspected. We remain perplexed by the lack of positive results for our competing principals model, and indeed the next obvious step in our research agenda is to understand the conditions under which MNAR non-response might lead to noticeably poorer estimates of ideal points based on observed data.

Relatedly, we plan to explore diagnostic methods to decide whether particular missingness patterns in roll-call data are likely to be non-ignorable (Zhang and Heitjan 2007). Contrary to work with missingness in survey data, where analysts have at their disposal a rather large number of covariates to build more sophisticated imputation models at least for MAR data, the main quandary in roll-call data is that we often lack relevant covariates at the individual level that might help us understand the incidence of non-response. The most important future step in our research, however, requires us to explore real legislatures to see if our methods are capable of recovering sensible ideological profiles. In this regard, we purport to build on the contribution of Clinton, Jackman and Rivers (2004*a*) to "most liberal Senator" debates. If indeed the bills that induce campaigning senators to fly back to Washington are different than those they are happy to miss, it is also possible that their observed vote choices on bills for which they are present are radically different from their unobserved vote choices on bills they miss. Such a pattern would induce the kind of MNAR abstention that these Clinton, Jackman and Rivers (2004*a*) hint at. As we move forward, we are aware that there remain formidable obstacles in the analysis of actual legislatures. Not only is it possible that several non-response processes are at play for different votes in any one legislative session, but the dynamics we alluded to at the beginning of the paper—misrepresentation of actual preferences (for example, through strategic voting on killer amendments), log-rolling, homogeneous vote choice induced by party discipline—are likely to have an effect both on patterns of non-response and on patterns of vote choice. These remain formidable tasks in the broader research program of recovering measures of ideology from observed votes.

A Appendix

A.1 Indifference model (Jags)

```
model { for (i in 1:n.legislators) {
    cut[i,1] <- cut[i,2]*(-1);
    for (j in 1:n.item) {
      y[i,j] ~ dcat(p[i,j,1:3]);
      p[i,j,1] <- Q[i,j,1];
      p[i,j,2] <- Q[i,j,2] - Q[i,j,1];
      p[i,j,3] <- 1 - Q[i,j,2];
      for(n in 1:2) { probit(Q[i,j,n]) <- cut[i,n] - beta[j]*theta[i] + alpha[j]; }
    }}
  # PRIORS
  for(j in 1:n.item) { alpha[j] ~ dnorm(0, 0.25); }
  for(j in 1:n.item) { beta[j]  ~ dnorm(0, 0.25); }
  theta[1] ~ dnorm(-2, 4);
  for(i in 2:(n.legislators-1)) { theta[i] ~ dnorm(0, 1); }
  theta[n.legislators] ~ dnorm(2, 4);
  for (i in 1:n.legislators) { cut[i,2] ~ dunif(0, 5); }}
```

A.2 Alienation model (Jags)

```
model { for(i in 1:n.legislators) {
    for(j in 1:n.item) {
      rc[i,j] ~ dbern(p[i,j]);
      m.voted[i,j] ~ dbern(q[i,j]);
      probit(p[i,j]) <- mu[i,j,1];
      probit(q[i,j]) <- mu[i,j,2];
      mu[i,j,1] <- beta[j]*theta[i]  - alpha[j];
      mu[i,j,2] <- lambda[1] - lambda[2]*theta.sq[i];
    }}
#  PRIORS
  for (j in 1:n.item) { alpha[j] ~ dnorm (0, 0.25); }
  for (j in 1:n.item) { beta[j]  ~ dnorm (0, 0.25); }
  lambda[1] ~ dnorm(1, 0.1)T(0,);
  lambda[2] ~ dnorm(1, 0.1)T(0,);
  theta[1] ~ dnorm(0, 1);
  theta[2] <- -1;
  for(i in 3:(n.legislators-2)) { theta[i] ~ dnorm(0, 1); }
  theta[n.legislators-1] <- 1;
  theta[n.legislators] ~ dnorm(0, 1);
  for(i in 1:n.legislators) { theta.sq[i] <- pow(theta[i],2); }}
```

A.3 Competing principals model (Winbugs)

```
model { for (i in 1:n.legislators)  {
     for (j in 1:n.item) {
        zstar[i,j,1:2] ~ dmnorm(mu[i,j,1:2],Tau.mu[,])I(low[i,j,1:2],upp[i,j,1:2])
```

```
        mu[i,j,1]  <- beta[j]*theta[i] - alpha[j]
        mu[i,j,2]  <- delta[1]*agree[i,j] + delta[2]
        agree[i,j] <- step(together[i,j])
        together[i,j] <- sign[i,j]*lead[party[i],j]
  sign[i,j] <- step(zstar[i,j,1])-0.5
     }}
# PRIORS
  Tau.mu[1:2,1:2] ~ dwish(R[,],2)
  for(j in 1:n.item){ alpha[j] ~ dnorm(0, 0.25) }
  for(j in 1:n.item){ beta[j]  ~ dnorm(0, 0.25) }
     delta[1] ~ dnorm(0, 0.25)
     delta[2] ~ dnorm(0, 0.25)
        theta[1] ~ dnorm(-2, 100)
    for(i in 2:(I-3))
    {
        theta[i] ~ dnorm(0, 1)
    }
        theta[I-2] ~ dnorm( 2, 100)
}
```

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